MATH 332
Quiz V
April 13, 1988

Name Key

1. Is (1,2,-1,2), (1,0,1,0), (0,1,0,2) and orthogonal set of vectors in R^2, using the ordinary Euclidean inner product? orthonormal? Justify.

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$(1,2,-1,2)\cdot(1,0,1,0) = 0$

$(1,0,1,0)\cdot(0,1,0,2) = 0$ NO

$(1,2,-1,2)\cdot(0,1,0,2) = 6$

2. Using the following inner product on P_2, are the polynomials x and x^2 orthogonal? Find $\|x\|$.

$\langle p,q\rangle = \int_{-1}^{1} p(x)q(x)\,dx$

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$\langle x, x^2\rangle = \int_{-1}^{1} x\,x^2\,dx = \int_{-1}^{1} x^3\,dx = \frac{x^4}{4}\Big|_{-1}^{1} = 0$ Yes

$\|x\|^2 = \langle x,x\rangle = \int_{-1}^{1} x^2\,dx = \frac{x^3}{3}\Big|_{-1}^{1} = \frac{1}{3}+\frac{1}{3} = \frac{2}{3}$ $\|x\| = \sqrt{\frac{2}{3}}$

3. Find an orthonormal basis for the space spanned by the vectors (1,0,0,0), (2,3,1,0), and (4,1,2,3).

basis - computed.

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$u_1 = (1,0,0,0) = v_1$

$u_2 = (2,3,1,0)$

$u_3 = (4,1,2,3)$

$v_2 = u_2 - \frac{u_2\cdot v_1}{v_1\cdot v_1} v_1 = (2,3,1,0) - \frac{2}{1}(1,0,0,0)$

$= (0,3,1,0)$

$v_3 = u_3 - \frac{u_3\cdot v_1}{v_1\cdot v_1} v_1 - \frac{u_3\cdot v_2}{v_2\cdot v_2} v_2$

$= (4,1,2,3) - \frac{4}{1}(1,0,0,0) - \frac{5}{10}(0,3,1,0)$

$= (4,1,2,3) - (4,0,0,0) - (0,3/2,1/2,0) = (0,-1/2,3/2,3)$ or $(0,-1,3,6)$

$v_1 = (1,0,0,0)$

$v_2 = \frac{1}{\sqrt{10}}(0,3,1,0)$

$v_3 = \frac{1}{\sqrt{46}}(0,-1,3,6)$

many got.

4. Find the projection of the vector (2,6,4) onto the plane spanned by (1,0,0) and (2,1,1). Then write (2,6,4) as the sum of a vector in this plane and one which is orthogonal to the plane.

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3 or 4 got all

$(1,0,0)\cdot(2,1,1) = 2 \neq 0$

u_1 u_2

v_1 $v_2 = (2,1,1) - \frac{2}{1}(1,0,0)$

$= (0,1,1)$

$\bar{p} = \frac{(2,6,4)\cdot(1,0,0)}{1}(1,0,0) + \frac{(2,6,4)(0,1,1)}{2}(0,1,1)$

$= (2,0,0) + 5(0,1,1) = (2,5,5)$

$\bar{x} - \bar{p} = (2,6,4) - (2,5,5) = (0,1,-1)$

$(2,6,4) = (2,5,5) + (0,1,-1)$