

1. Complete the following DEFINITION: The vectors v_1, \dots, v_n are linearly independent if and only if

(5)

2. Find a basis for the solution space of the following system of homogeneous equations. The solution space is an 2 dimensional subspace of \mathbb{R}^5 .

(7)

$$\begin{bmatrix} 1 & 2 & 0 & 1 & -2 & | & 0 \\ 0 & 2 & 1 & 3 & 1 & | & 0 \\ 0 & 0 & -1 & 2 & 1 & | & 0 \end{bmatrix}$$

$$\begin{aligned} x_5 &= t & x_4 &= s \\ -x_3 + 2s + t &= 0 \\ x_3 &= 2s + t \end{aligned}$$

$$2x_2 + (2s+t) + 3s + t = 0$$

$$2x_2 = -5s - 2t$$

$$x_2 = -\frac{5}{2}s - t$$

$$x_1 + (-5s - 2t) + s - 2t = 0$$

$$x_1 = 4s + 4t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = s \begin{bmatrix} 4 \\ -5/2 \\ 2 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 4 \\ -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

3. What is the rank of

(3)

$$\begin{bmatrix} 2 & 9 \\ 0 & 3 \end{bmatrix} \quad 2$$

4. Consider the following matrix:

(5)

$$\begin{bmatrix} 1 & 2 & 7 & 6 & -1 & 2 & 0 \\ 0 & 0 & 1 & 9 & 3 & 0 & 2 \\ 0 & 0 & 0 & 2 & 9 & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

a. The row space is an 3 dimensional subspace of \mathbb{R}^7 .

b. The column space is an 3 dimensional subspace of \mathbb{R}^7 .

c. The rank is 3.

5. Do the vectors $(1,2,3)$, $(1,1,2)$, and $(2,0,2)$ form a basis for \mathbb{R}^3 ? Justify.

(5)

$$\det \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 2 & 0 & 2 \end{bmatrix} = -2 \begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix} = -2(2-4) + (2-6) = 4-4 = 0$$

NO!

(OR) $\begin{bmatrix} 1 & 1 & 2 \\ 0 & -1 & -4 \\ 0 & 0 & 0 \end{bmatrix}$

6. Give a basis for the subspace of symmetric matrices in M_{22} . What is the dimension of this subspace? What is the dimension of M_{22} ?

(5)

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

dim 3