Fund h/f @ 12
Half @ 20
One said it was "easy"

MATH 332
Quiz V
November 14, 1990 Name Key

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(5) 1. Complete the following DEFinition: The vectors $v_1, \ldots, v_n$ are linearly independent if and only if

6 2. Are the vectors (1,-1,2 0), (3,-1,6,6), and (2,-3,4,-3) linearly independent? Justify.

$k_1(1,-1,3,0)+k_2(3,-1,6,6)+k_3(2,-3,4,-3)=(0,0,0,0)$

$$\begin{bmatrix} 1 & 3 & 2 \\ -1 & -1 & -3 \\ 2 & 6 & 4 \\ 0 & 6 & -3 \end{bmatrix}\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$X_3 = t$ so soln - so dep

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$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 3 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(7) 3. Find a basis for the solution space of the following system of homogeneous equations. The solution space is an 2 dimensional subspace of $\mathbb{R}^5$.

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$$\left[\begin{array}{ccccc|c} 1 & 2 & 0 & 1 & -2 & 0 \\ 0 & 2 & 1 & 3 & 1 & 0 \\ 0 & 0 & -1 & 2 & 1 & 0 \end{array}\right]$$

$X_5 = t$

$X_4 = s$

$-X_3 + 2s + t = 0$

$X_3 = 2s + t$

$2X_2 + (2s+t) + 3s + t = 0$

$2X_2 = -5s - 2t$

$X_2 = -\frac{5}{2}s - t$

$X_1 + (-5s - 2t) + s - 2t = 0$

$X_1 = \cancel{4t}\ 4s + 4t$

$s(4, -\frac{5}{2}, 2, 1, 0) + t(4, -1, 1, 0, 1)$

(6) 4. Give a basis for the subspace of matrices in M_{22} of the form: $\begin{bmatrix} a & 0 \\ a & b \end{bmatrix}$, where a and b are any real numbers.
What is the dimension of this subspace? What is the dimension of M_{22}?

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$\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

dim = 2, dim M_{22} = 4

(7) 5. Prove: If $v_1, \ldots, v_n$ are linearly dependent, then at least one of the v_i can be written as a linear combination of the others.

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If dep then there is a set of $k_1, \ldots, k_n$ not all zero s.t. $k_1 v_1 + \cdots + k_n v_n = 0$.

assume wolog that $k_1 \neq 0$ then

$k_1 v_1 = -k_2 v_2 - \cdots - k_n v_n$

$v_1 = -\frac{k_2}{k_1} v_2 - \frac{k_3}{k_1} v_3 - \cdots - \frac{k_n}{k_1} v_n$ since $k_1 \neq 0$.