Time OK
$\bar{x}$ 26.8/30
m 28

attend 23m
First @ 9Y, 14, 15
labs left @ 15-16
all but 4 by 20
all by 25

MATH 332
Quiz V
November 8, 1993

Name Key

1. Find the least squares line, $y = at + b$, for the data:

t	-2	-1	0	1	2
y	-4.9	-2.9	-1.2	1.1	2.8

6

$$A = \begin{bmatrix} -2 & 1 \\ -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} \quad A^tA = \begin{bmatrix} -2 & -1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & 5 \end{bmatrix}$$

$$A^ty = \begin{bmatrix} -2 & -1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -4.9 \\ -2.9 \\ -1.2 \\ 1.1 \\ 2.8 \end{bmatrix} = \begin{bmatrix} 19.4 \\ -5.1 \end{bmatrix}$$

$$\begin{bmatrix} 10 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 19.4 \\ -5.1 \end{bmatrix}$$

$a = 1.94 \quad b = -1.02$

$y = 1.94x - 1.02$

8
must close

2. Calculate the determinant of each of the following:

a. $\begin{bmatrix} 2 & 1 & 0 \\ 7 & 1 & 2 \\ 2 & -1 & 3 \end{bmatrix}$

$$\det = 2\begin{vmatrix} 1 & 2 \\ -1 & 3 \end{vmatrix} - 1\begin{vmatrix} 7 & 2 \\ 2 & 3 \end{vmatrix}$$

$= 2(3+2) - (21-4) = 10 - 17 = -7$

all but 1 (sign)

b. $\begin{bmatrix} 2 & 2 & 2 & -5 \\ 0 & 0 & 2 & 3 \\ 0 & -3 & 3 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

$2 \det \begin{bmatrix} 0 & 2 & 3 \\ -3 & 3 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ or $2 \cdot 3 \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$

$= 2 \cdot 2 \begin{vmatrix} 0 & 2 \\ -3 & 3 \end{vmatrix} = 4(6) = 24$

must (may wrap)

c. $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 2 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$

$6(-4) = -24$

all but 1
2 multiplies

3. Is the following matrix invertible? Show work.

$\begin{bmatrix} 2 & -1 & 3 \\ 7 & 1 & 2 \\ 2 & -1 & 3 \end{bmatrix}$ $\det = 0$ no.

several got $\det = 0$ but said yes

large size

4. Find the eigenvalues and corresponding eigenvectors for

$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ $\begin{vmatrix} 2-\lambda & 1 \\ 3 & 4-\lambda \end{vmatrix} \rightarrow (2-\lambda)(4-\lambda) - 3$

$8 - 6\lambda + \lambda^2 - 3 = 0$

$\lambda^2 - 6\lambda + 5 = 0$

$(\lambda - 5)(\lambda - 1)$

$\lambda = 1$

$\begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$x_2 = t$

$x_1 = -t$

$(-1, 1)$

$\lambda = 5$

$\begin{bmatrix} -3 & 1 \\ 3 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} -3 & 1 \\ 0 & 0 \end{bmatrix}$

$x_2 = t \quad x_1 = \frac{1}{3}t$

$(\frac{1}{3}, 1)$ or $(1, 3)$