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MATH 332
Quiz V
November 1, 1989 Name Key Answers

(5) 1. Complete the following DEFinition: The vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$ are linearly independent if and only if

(7) 2. Find a basis for the solution space of the following system of homogeneous equations. The solution space is an 2 dimensional subspace of $\mathbb{R}^5$.

$$\left[\begin{array}{ccccc|c} 1 & 2 & 0 & 1 & -2 & 0 \\ 0 & 2 & 1 & 3 & 1 & 0 \\ 0 & 0 & -1 & 2 & 1 & 0 \end{array}\right]$$

$x_5 = t, \quad x_4 = s$

$-x_3 + 2s + t = 0$

$x_3 = 2s + t$

$2x_2 + (2s+t) + 3s + t = 0$

$2x_2 = -5s - 2t$

$x_2 = -\frac{5}{2}s - t$

$x_1 + (-5s - 2t) + s - 2t = 0$

$x_1 = 4s + 4t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = s\begin{bmatrix} 4 \\ -5/2 \\ 2 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 4 \\ -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

basis

$(4, -5/2, 2, 1, 0), (4, -1, 1, 0, 1)$

(3) 3. What is the rank of

$$\begin{bmatrix} 2 & 9 \\ 0 & 3 \end{bmatrix} \qquad 2$$

(5) 4. Consider the following matix:

$$\begin{bmatrix} 1 & 2 & 7 & 6 & -1 & 2 & 0 \\ 0 & 0 & 1 & 9 & 3 & 0 & 2 \\ 0 & 0 & 0 & 2 & 9 & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

a. The row space is an 3 dimensional subspace of $\mathbb{R}^7$.

b. The column space is an 3 dimensional subspace of $\mathbb{R}^4$.

c. The rank is 3.

(5) 5. Do the vectors (1,2,3), (1,1,2), and (2,0,2) form a basis for R^3? Justify.

$$\det\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 2 & 0 & 2 \end{bmatrix} = -2\begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix} = -2(-2) + 2 - 6 = 0$$

not lin indep (rank < 3)

so not a basis for $\mathbb{R}^3$

(This is one way.)

(5) 6. Give a basis for the subspace of symmetric matrices in M_{22}. What is th dimension of this subspace? What is the dimension of M_{22}?

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \qquad \dim = 3$$

$\dim M_{22} = 4$