Time OK

MATH 332
Quiz VI
April 27, 1988

Name KEY

1. Consider the linear transformation $F(x,y) = (2x-y, x, x+y)$.
   a. Is (1,2) in the kernel of F?
   b. Write the standard matrix for this transformation.

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(a) $F(1,2) = (0, 1, 3)$ no!

(b) $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$

2. For the transformation

$$T(x) = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 7 & -4 \end{bmatrix} x$$

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a. T is a linear transformation of R^{3} to R^{2} (fill in).
b. Find the rank and nullity of T.
c. Find a basis for the null space of T.

(b) $\begin{bmatrix} 1 & 2 & -1 \\ 3 & 7 & -4 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \end{bmatrix}$

rank = 2
nullity = 1

(c) $x_3 = t$
$x_2 = t$
$x_1 + 2t - t = 0$
$x_1 = -t$

$(-1, 1, 1)$

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3. Consider the function $F: P_2 \to P_2$ given by $F(a_0 + a_1x + a_2x^2) = a_0x + a_1x^2$.
   a. Compute $F(2 + x + 3x^2)$.
   b. Show that F is a linear transformation?

(a) $F(2 + x + 3x^2) = 2x + x^2$

(b) $F((a_0 + a_1x + a_2x^2) + (b_0 + b_1x + b_2x^2))$
$= a_0 + b_0 + (a_1 + b_1)x^2$
$= F(a_0 + a_1x + a_2x^2) + F(b_0 + b_1x + b_2x^2)$

$F(k(a_0 + a_1x + a_2x^2)) = ka_0x + ka_1x^2$
$= k(a_0x + a_1x^2)$
$= kF(a_0 + a_1x + a_2x^2)$

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4. Find the eigenvalues of the matrix

$\begin{bmatrix} -2 & 3 \\ 0 & 4 \end{bmatrix}$ $\det(\lambda I - A) = 0$ $\det \begin{bmatrix} \lambda+2 & -3 \\ 0 & \lambda-4 \end{bmatrix} = 0$

$(\lambda + 2)(\lambda - 4) - 0 = 0$
$\lambda = -2, 4$