MATH 332
Quiz VI
November 17, 1989 Name KEY

$\bar{x} = 22.5/30$
med 24

1\. Complete the following DEFinitions: 10

a. The <u>dimension</u> of a vector space is

b. A function F from a vector space V to a vector space W is called a <u>linear transformation</u> if and only if

2\. Prove: If $v_1, v_2, \ldots, v_n$ form a basis for a vector space V, then any vector v can be written as a linear combination of the basis vectors in only one way. 6

Suppose $v = a_1 v_1 + \cdots + a_n v_n$
and $v = b_1 v_1 + \cdots + b_n v_n$

then $a_1 v_1 + a_2 v_2 + \cdots + a_n v_n = b_1 v_1 + \cdots + b_n v_n$

$(a_1 - b_1) v_1 + (a_2 - b_2) v_2 + \cdots + (a_n - b_n) v_n = 0$

since $v_1, \ldots, v_n$ are lin indep

$a_1 - b_1 = 0, \ldots, a_n - b_n = 0$ or $a_1 = b_1, \ldots, a_n = b_n$

3\. Find the equation of the least squares straight line for the data: 6

x	-4	-1	1	2	3
y	-6.8	-.9	2.9	5.8	6.6

$y = b + mx$

What would change if we are fitting the curve $y = \alpha + \beta x + \gamma x^2$? (Set up, do NOT solve.)

$$M = \begin{bmatrix} 1 & -4 \\ 1 & -1 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \quad Y = \begin{bmatrix} -6.8 \\ -.9 \\ 2.9 \\ 5.8 \\ 6.6 \end{bmatrix}$$

$$M^t M = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -4 & -1 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -4 \\ 1 & -1 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 5 & \cancel{0} \\ 0 & 31 \end{bmatrix}$$

$$M^t Y = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -4 & -1 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} -6.8 \\ -.9 \\ 2.9 \\ 5.8 \\ 6.6 \end{bmatrix} = \begin{bmatrix} 7.6 \\ 62.4 \end{bmatrix}$$

$$\begin{bmatrix} 5 & \cancel{0} \\ \cancel{0} & 31 \end{bmatrix} \begin{bmatrix} b \\ m \end{bmatrix} = \begin{bmatrix} 7.6 \\ 62.4 \end{bmatrix}$$

$b = 1.52$
$\cancel{m = 2.01}$

$y = 1.12 + 1.98x$
$\cancel{y = 2.01x + 1.52}$

$$\frac{1}{\cancel{155}-1} \begin{bmatrix} 31 & -1 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} 7.6 \\ 62.4 \end{bmatrix}$$

$$\frac{1}{\cancel{154}} \begin{bmatrix} 173.2 \\ 304.4 \end{bmatrix} = \begin{bmatrix} 1.059 \\ 1.2 \end{bmatrix}$$

$$\begin{bmatrix} 1.12 \\ \cancel{2.01} \\ 1.98 \end{bmatrix}$$

quadratic case

$$M = \begin{bmatrix} 1 & -4 & 16 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix}$$

4\. Is the following matrix orthogonal? Show work. 3

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\cancel{\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 2 \end{bmatrix}}$$ no.

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

5\. The row space of 5

$$\begin{bmatrix} 1 & 1 & 0 & 2 & 4 & 0 \\ 0 & 0 & 0 & 3 & 0 & 5 \\ 0 & 0 & 0 & 0 & 2 & 1 \end{bmatrix}$$

is a <u>3</u> dimensional subspace of <u>$\mathbb{R}^6$</u>.

Give a basis for the column space.

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 5 \\ 1 \end{bmatrix} \quad \begin{bmatrix} -4 \\ 0 \\ 2 \end{bmatrix}$$

← must do row reduction on A^t (or rank = 3)