1

Time OK

allowed 22 min
First left @ :16
Many leaving @ 15 or 19
1/2 left @ 20

MATH 332
Quiz VI
November 28, 1990 Name Key

8 1. Complete the following DEFinitions:

a. The rank of a matrix is the dim of row space = (col sp) 20/25

b. A function F from a vector space V to a vector space W is called a linear transformation if and only if

$F(\underline{u}+\underline{v}) = F(\underline{u}) + F(\underline{v})$ for all $\underline{u}, \underline{v}$ in V

$F(k\underline{u}) = k\,F(\underline{u})$ " " $\underline{u}$ in V, all reals k 17/25

5 2. For the matrix

$$\begin{bmatrix} 1 & 1 & 2 & 2 & -4 & 0 \\ 0 & 0 & 0 & 3 & 0 & 5 \\ 0 & 0 & 0 & 0 & 2 & 1 \end{bmatrix}$$

9

a. The row space is a 3 dimensional subspace of $\mathbb{R}^6$.

b. The column space is a 3 dimensional subspace of $\mathbb{R}^3$.

c. The rank is 3.

5 3. Is the vector (2,2,11) in the column space of the following? Give reasons.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 0 \\ 2 & 10 & 6 \end{bmatrix} \qquad \left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 2 & 0 & 2 \\ 2 & 10 & 6 & 11 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 6 & 0 & 7 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

NO 15

6 4. Find a basis for the space spanned by the vectors (1,2,2,1,-3), (-4,-6,-11,-3,13), (2,2,7,1,-7). This is a ______ dimensional subspace of ______.

see other sheet

Put on final! some CS $(\frac{1}{2})$ some rel space $(\frac{1}{2})$ 8

6 5. Prove: If $v_1, v_2, \ldots, v_n$ form a basis for a vector space V, then any vector v in V can be written as a linear combination of the basis vectors in only one way.

Suppose $\underline{v} = k_1\underline{v}_1 + \cdots + k_n\underline{v}_n$ and $\underline{v} = l_1\underline{v}_1 + \cdots + l_n\underline{v}_n$ 18

$k_1\underline{v}_1 + \cdots + k_n\underline{v}_n = l_1\underline{v}_1 + \cdots + l_n\underline{v}_n$

$(k_1 - l_1)\underline{v}_1 + \cdots + (k_n - l_n)\underline{v}_n = \underline{0}$

so since $\underline{v}_1, \ldots, \underline{v}_n$ lin indep,

$k_1 - l_1 = 0, \ldots, k_n - l_n = 0$

or $k_1 = l_1, \ldots, k_n = l_n$.