Time ok

$\bar{x} = 23.8/30$

$m = 23.5$

all were @ 4 min

started to leave @ 15 (several)

mod = 20

all by 25

MATH 332
Quiz VI
November 19, 1993

Name Key

1. Give a complete definition: The matrices A and B are <u>similar</u>

(6)

10

2. For z = 2 + 3i, and w = 3 - 2i, find

6

a. zw $\quad (2+3i)(3-2i) = 6 + 9i - \cancel{4i} - 6i^2 = 6 + 5i + 6 = 12+5i$

7 missed

b. $\bar{z}$ + w $\quad (2-3i) + (3-2i) = 5-5i$

most

3. Find the eigenvalues of

6

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 5 \\ 0 & -1 & 0 \end{bmatrix} \quad \det \begin{bmatrix} 1-\lambda & 2 & 3 \\ 0 & -2-\lambda & 5 \\ 0 & -1 & -\lambda \end{bmatrix} = (1-\lambda)\left[(-2-\lambda)(-\lambda)+5\right]$$

$$= (1-\lambda)(+2\lambda + \lambda^2 + 5)$$

$$= (1-\lambda)(\lambda^2 + 2\lambda + 5)$$

$$\lambda = 1 \qquad \lambda = \frac{-2 \pm \sqrt{4-20}}{2} = -1 \pm 2i$$

6

4 used comp on it

some couldn't factor

4. Diagonalize A by writing it as SDS^{-1}, where D is a diagonal matrix.

6

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix} \qquad \lambda = 1, 4 \qquad S = \begin{bmatrix} \cancel{2/3} & 1 & \frac{2}{3} \\ 1 & 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$$

$$\lambda = 1 \quad \begin{bmatrix} 0 & 2 \\ 0 & 3 \end{bmatrix} \to \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$$

$x_2 = 0$, $x_1 = t$ $\quad \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$A = \begin{bmatrix} \cancel{2/3}\, 1 & 2/3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & -2/3 \\ 0 & 1 \end{bmatrix}$$

7

$$\lambda = 4 \quad \begin{bmatrix} -3 & 2 \\ 0 & 0 \end{bmatrix} \qquad x_2 = t, \quad -3x_1 + 2t = 0, \quad x_1 = \frac{2}{3}t$$

$$S^{-1} = \begin{bmatrix} 1 & -2/3 \\ 0 & 1 \end{bmatrix}$$

most messed up 1 vector

$$\begin{bmatrix} 2/3 \\ \cancel{0} \end{bmatrix} \text{ or } \begin{bmatrix} 2 \\ 3 \end{bmatrix} \qquad \text{or} \quad \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \frac{1}{3} \begin{bmatrix} 3 & -2 \\ 0 & \cancel{1} \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2/3 \\ 0 & 1/3 \end{bmatrix}$$

5. Prove a. or b.

a. The eigenvectors for distinct eigenvalues of a matrix are linearly independent

b. A and A^T have the same eigenvalues.

(5) got a 4 more close

b. $(A - \lambda I)^T = A^T - \lambda I$

$\det (A - \lambda I)^T = \det(A - \lambda I)$

so $\det (A - \lambda I) = \det (A^T - \lambda I) = 0$

same same poly, same roots.

so same e.v.

same det A^{-1}, gave 3 pts

a. $a_1 x_1 + a_2 x_2 = 0$

$A x_1 = \lambda_1 x_1$

$A x_2 = \lambda_2 x_2$

$x_1 \neq 0$, $x_2 \neq 0$

$A(a_1 x_1 + a_2 x_2) = A\,0$

$a_1 A x_1 + a_2 A x_2 = 0$

$a_1 \lambda_1 x_1 + a_2 \lambda_2 x_2 = 0$

also $\lambda_1 (a_1 x_1 + a_2 x_2) = \lambda_1 \cdot 0$

$a_1 \lambda_1 x_1 + a_2 \lambda_1 x_2 = 0$

Sub $\quad a_2 \lambda_2 x_2 - a_2 \lambda_1 x_2 = 0$

$\Rightarrow (\lambda_2 - \lambda_1)(a_2) x_2 = 0$

$\lambda_2 \neq \lambda_1$, $x_2 \neq 0 \Rightarrow a_2 = 0$