med 19
$\bar{x}$ 12.5

Time OK
allowed extra time
First left at :20
Most at :30

MATH 332
Quiz VII
December 4, 1989

Name: ~~Answer~~ Key

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1.Complete the following DEFinition: The <u>range</u> of the linear transformation T: V → W is set of {y in W s.t. T(x)=y for some x in V.}

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2.Complete and prove: The nullspace (kernel) of the linear transformation T: V → W is a subspace of V.

$K = \{x \mid T(x) = 0\}$

(1) x, y in K
Show $x+y$ in K
i.e. $T(x+y) = 0$

$T(x+y) = T(x) + T(y)$
$= 0 + 0$ since in K
$= 0$

(2) closure of scalar mult
$x \in K$ show kx in K
for any k real
i.e. $T(k\bar{x}) = \bar{0}$.

$T(k\bar{x}) = kT(\bar{x})$
$= k \cdot 0 = 0$
so since closed, must be a subspace

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3.For the linear transformation T((x,y,z)) = (2x,y + z,2x - z)

a. Find T((2,1,-3)) $= (4, -2, 7)$

b. Write in matrix form T(x) = Ax.

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 2 & 0 & -1 \end{bmatrix}$$

c. Find the kernel (null space) of T. Show work, or give reasons! (Think!)

rank = 3, so ker = $\{(0,0,0)\}$

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4.Let T: $R^4 \to R^6$ be given by T(x) = Ax for some matrix A with rank 3.

6×4 · 4×1

a. The matrix A is 6 × 4.

b. The range is a 3 dimensional subspace of $\mathbb{R}^6$.

c. The nullspace (kernel) is a 1 dimensional subspace of $\mathbb{R}^4$.

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5.Suppose that T((2,3)) = (1,3) and T((0,2)) = (2,-2). What is T((2,5))?

$a(2,3) + b(0,2) = (2,5)$

$a = 1,\ b = 1$

$(2,3) + (0,2) = (2,5)$

$T(2,3) + T(0,2) = (1,3) + (2,-2)$

$= (3,1)$

$\frac{1}{3}$