

1. Complete the following DEFINITION: The range of the linear transformation  $T: V \rightarrow W$  is set of  $\{y \text{ in } W \text{ s.t. } T(x)=y \text{ for some } x \text{ in } V.\}$

2. Complete and prove: The nullspace (kernel) of the linear transformation  $T: V \rightarrow W$  is a subspace of  $V$ .

$K = \{x \mid T(x) = 0\}$

(1)  $x, y \in K$   
 Show  $x+y \in K$   
 i.e.  $T(x+y) = 0$

$T(x+y) = T(x) + T(y)$   
 $= 0 + 0$  since  $x, y \in K$   
 $= 0$

(2) closure of scalar mult  
 $x \in K$  show  $kx \in K$   
 for any  $k$  real  
 i.e.  $T(kx) = 0$

$T(kx) = kT(x)$   
 $= k \cdot 0 = 0$   
 so subspace closed, must be a subspace

3. For the linear transformation  $T((x,y,z)) = (2x, y+z, 2x-z)$

a. Find  $T((2,1,-3)) = (4, -2, 7)$

b. Write in matrix form  $T(x) = Ax$ .

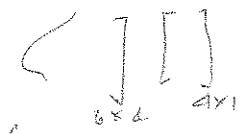
$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 2 & 0 & -1 \end{bmatrix}$$

c. Find the kernel (null space) of  $T$ . Show work, or give reasons! (Think!)

rank = 3, so  $\ker = \{(0,0,0)\}$

4. Let  $T: \mathbb{R}^4 \rightarrow \mathbb{R}^6$  be given by  $T(x) = Ax$  for some matrix  $A$  with rank 3.

a. The matrix  $A$  is 6 x 4.



b. The range is a 3 dimensional subspace of  $\mathbb{R}^6$ .

c. The nullspace (kernel) is a 1 dimensional subspace of  $\mathbb{R}^4$ .

5. Suppose that  $T((2,3)) = (1,3)$  and  $T((0,2)) = (2,-2)$ . What is  $T((2,5))$ ?

$a(2,3) + b(0,2) = (2,5)$

$a=1, b=1$

$(2,3) + (0,2) = (2,5)$

$T(2,3) + T(0,2) = (1,3) + (2,-2)$   
 $= (3,1)$

$\frac{1}{3}$