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MATH 332
Quiz VII
December 10, 1990

Allowed :21
Started no more @ :14
Time OK 15/26 @:20

Name Key

1. Complete the following DEFinition: The rank of the linear transformation
$T: V \to W$ is dimension of the range space.

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2. For the linear transformation $T((x,y,z)) = (x+2y+z, -3x-5y-11z)$: (2)

2 a. This is a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$.

2 b. Find $T((-2,1,-3)) =$ $(-3, 34)$

2 c. Written in matrix form $T(x) = Ax$, $A =$ $\begin{bmatrix} 1 & 2 & 1 \\ -3 & -5 & -11 \end{bmatrix}$

1 d. What is the rank of T? 2

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2 e. Is (2,3,-2) in the kernel (null space) of T?

$T(2,3,-2) = (6, 1) \neq (0,0)$ NO.

3 f. Is (13,-5) in the range of T?

$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 13 \\ -3 & -5 & -11 & -5 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 2 & 1 & 13 \\ 0 & 1 & -8 & 34 \end{array}\right]$ ∞ sols

Yes

3. Show that $T: M_{22} \to R$ is a linear transformation, where T is defined by:
$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = b + c$

$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix}\right) = T\begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix} = b+f+c+g$ must

$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) + T\left(\begin{bmatrix} e & f \\ g & h \end{bmatrix}\right) = b+c+f+g$

$T\left(k\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = T\left(\begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}\right) = kb + kc$

$kT\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = k[b+c] = kb + kc$

7. 4. Complete and prove: The nullspace (kernel) of the linear transformation
$T: V \to W$ is a subspace of V. hold

If $\underline{u}, \underline{v}$ in $\ker(T)$, $T(\underline{u}) = \underline{0}$ and $T(\underline{v}) = \underline{0}$

$T(\underline{u}+\underline{v}) = T(\underline{u}) + T(\underline{v}) = \underline{0} + \underline{0} = \underline{0}$ so $\underline{u+v}$ in $\ker(T)$

If $\underline{u}$ in $\ker(T)$, k any real; $T(\underline{u}) = \underline{0}$

$T(k\underline{u}) = kT(\underline{u}) = k\underline{0} = \underline{0}$ so $k\underline{u}$ in $\ker(T)$

Since closed under + and scalar mult,
$\ker(T)$ is a subspace of V.