

(12) 1. Find each determinant. Look for short cuts. Show calculations or give reasons.

$$(a) \begin{vmatrix} 1 & 1 & 2 & 1 \\ 2 & 2 & 4 & 2 \\ 3 & 1 & 6 & 3 \\ 2 & 1 & 1 & -1 \end{vmatrix} = 0 \text{ prop rows}$$

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 1 1/2 hrs  
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 2 hrs  
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 time OK*

$$(b) \begin{vmatrix} 1 & 0 & 1 & 5 \\ 1 & 0 & 3 & 4 \\ 2 & 0 & 3 & 3 \\ -1 & 0 & 6 & 2 \end{vmatrix} = 0 \text{ col of zeros}$$

$$(c) \begin{vmatrix} 2 & 1 & 2 & 5 \\ 0 & 0 & 2 & 8 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 3 & 5 \end{vmatrix} = 2 \begin{vmatrix} 0 & 2 & 1 \\ 2 & 1 & 1 \\ 0 & 3 & 5 \end{vmatrix} = 2(-2) \begin{vmatrix} 2 & 1 \\ 3 & 5 \end{vmatrix} = -4(3) = -12$$

$$-4(-12) = 48$$

(13) 2. If  $\det A = -5$ ,  $A$  is  $4 \times 4$ , find:

(a)  $\det A^{-1} = -\frac{1}{5}$

(b)  $\det 3A = 3^4(-5) = -405$

(c)  $\det A^T = -5$

(d)  $\det(AA^T) = (-5)^2 = 25$

(14) 3. Which of the following are true? Correct if not true.

$$\begin{vmatrix} 1 & 1 & 2 & -1 \\ 2 & 1 & 1 & 3 \\ 3 & 1 & 1 & 4 \\ 4 & 2 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 1 & 3 \\ 1 & 1 & 1 & 4 \\ 3 & 1 & 1 & 4 \\ 4 & 2 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 1 & 3 \\ 1 & 1 & 1 & 4 \\ 3 & 1 & 1 & 4 \\ 4 & 2 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 2 & 4 & -2 \\ 4 & 2 & 2 & 6 \\ 3 & 1 & 1 & 4 \\ 4 & 2 & 2 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 2 & 3 & 1 \\ 0 & 2 & 5 & 1 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 \\ 1 & 1 & -1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{vmatrix} = 0$$

*True*

(5) 4. Fill in the matrix to get the desired product.

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 & 2 \\ 2 & 1 & 3 & 6 \\ 1 & 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 0 & 5 \\ 2 & 1 & 3 & 6 \\ 1 & 2 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(10) 5. Solve the following system of equations by Gaussian elimination.

$$2x_1 + 5x_2 + x_3 + 4x_4 = 1$$

$$x_1 + 2x_2 + x_3 + 2x_4 = 1$$

$$4x_1 + 10x_2 + 3x_3 + 11x_4 = 4$$

$$\left[ \begin{array}{cccc|c} 2 & 5 & 1 & 4 & 1 \\ 1 & 2 & 1 & 2 & 1 \\ 4 & 10 & 3 & 11 & 4 \end{array} \right] \rightarrow \left[ \begin{array}{cccc|c} 1 & 2 & 1 & 2 & 1 \\ 2 & 5 & 1 & 4 & 1 \\ 4 & 10 & 3 & 11 & 4 \end{array} \right] \rightarrow \left[ \begin{array}{cccc|c} 1 & 2 & 1 & 2 & 1 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 2 & 1 & 3 & 0 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} 1 & 2 & 1 & 2 & 1 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & 1 & 3 & 2 \end{array} \right] \rightarrow \left[ \begin{array}{cccc|c} 1 & 2 & 1 & 2 & 1 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & 0 & 3 & 1 \end{array} \right]$$

$$\begin{aligned} x_4 &= t \\ x_3 &= 2 - 3t \\ x_2 &= -1 + 2 - 3t \\ &= 1 - 3t \\ x_1 &= 1 - 2t - (2 - 3t) \\ &= -2(1 - 3t) \end{aligned}$$

(5) 6. Is the matrix  $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 0 \\ 1 & -1 & 0 \end{bmatrix}$  invertible?

$$\begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix} = 2 \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = 2(-3) \neq 0$$

Yes.

$$\begin{aligned} x_1 &= -3 + 7t \\ x_2 &= 1 - 3t \\ x_3 &= 2 - 3t \\ x_4 &= t \end{aligned}$$

$$\left[ \begin{array}{cccc|c} 1 & 2 & 0 & 4 & -1 \\ 0 & 1 & 0 & 3 & 1 \\ 0 & 0 & 1 & 3 & 2 \end{array} \right] \quad \left[ \begin{array}{cccc|c} 1 & 0 & 0 & -7 & -3 \\ 0 & 1 & 0 & 3 & 1 \\ 0 & 0 & 1 & 3 & 2 \end{array} \right]$$

$$\begin{aligned} x_4 &= t \\ x_3 &= 2 - 3t \\ x_2 &= 1 - 3t \\ x_1 &= -3 + 7t \end{aligned}$$

(12) 7. Systems of equations have been reduced to the following. Find all solutions of each.

$$\begin{bmatrix} -2-t \\ 2-t \\ t \\ 2-s-5t \\ s \\ -2t \\ t \end{bmatrix}$$

(a)  $\left[ \begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right]$

$$x_3 = t \quad x_2 = 2-t$$

$$x_1 = -2t - (2-t) = -2-t$$

(b)  $\left[ \begin{array}{cccc|c} 1 & 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$

$$x_4 = t \quad x_3 = -2t$$

$$x_2 = s \quad x_1 = 2 - 2(2-t) - t$$

$$= 2 - s - 2(-2t) - t$$

$$= 2 - s - 5t$$

(c)  $\left[ \begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{array} \right]$

$\emptyset$

(5) 8. Does the system of equations

$$x_1 + 2x_2 + x_3 = 0$$

$$x_1 - x_2 + x_3 = 0$$

$$x_1 - 2x_2 - 3x_3 = 0$$

have solutions other than  $x_1 = 0, x_2 = 0, x_3 = 0$ ? Show work.

$$\det \begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ 1 & -2 & -3 \end{bmatrix} \begin{matrix} 12 \\ -1 \\ 1-2 \end{matrix} = 3 + (-2) - (-1) - (-2) - (-3) > 0$$

NO.

(6) 9. Find the equation of the plane in  $\mathbb{R}^3$  spanned by the vectors  $(1, 2, 1)$  and  $(2, 1, -1)$ .

$$\begin{vmatrix} x & y & z \\ 1 & 2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} i - \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} j + \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} k$$

$$= (3, -3, 3) \quad 3(x-1) - 3(y-2) + 3(z-1) = 0$$

$$\boxed{3x - 3y + 3z = 0}$$

or  $x - y + z = 0$

(5) 10. Show that if  $AB = I$  and  $CA = I$ , then  $B = C$ .

$$AB = I$$

$$C(AB) = CI = C$$

$$(CA)B = C$$

$$IB = C$$

$$B = C$$

$$A^{-1} = B$$

(6) 11. Find the orthogonal projection of the vector  $(2, 5)$  onto the vector  $(-3, -4)$ .

$$\frac{(2, 5) \cdot (-3, -4)}{\|(-3, -4)\|^2} (-3, -4) = \frac{-6 - 20}{9 + 16} (-3, -4)$$

$$= -\frac{26}{25} (-3, -4) = \left(\frac{78}{25}, \frac{104}{25}\right)$$

(12) 12. Let  $\vec{u} = (1, 2, 3, -1)$  and  $\vec{v} = (2, -1, 0, 2)$ . Find

(a)  $\vec{u} \cdot \vec{v} = 1 \cdot 2 + 2(-1) + 0 - 2 = -2$

(b)  $3\vec{u} + \vec{v} = (3, 6, 9, -3) + (2, -1, 0, 2) = (5, 5, 9, -1)$

(c)  $\|\vec{u}\| = \sqrt{1^2 + 2^2 + 3^2 + 1^2} = \sqrt{15}$

(d) distance between  $\vec{u}$  and  $\vec{v} = \sqrt{1^2 + 3^2 + 3^2 + 3^2} = \sqrt{28} = 2\sqrt{7}$

(12) 13. Find the matrix for each of the following linear transformations.

(a)  $T(x_1, x_2) = (x_1 + x_2, 2x_2 - x_1)$

$$T(1, 0) = (1, -1)$$

$$T(0, 1) = (1, 2)$$

$$\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}$$

(b)  $T(x_1, x_2, x_3) = (2x_1, x_2 + x_3 - 2x_1)$

$$T(1, 0, 0) = (2, -2)$$

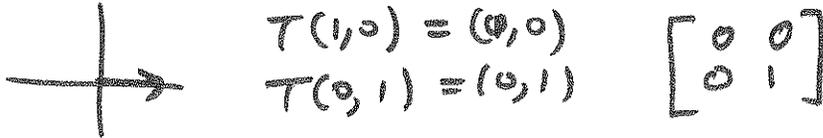
$$T(0, 1, 0) = (0, 1)$$

$$T(0, 0, 1) = (0, 1)$$

$$\begin{bmatrix} 2 & 0 & 0 \\ -2 & 1 & 1 \end{bmatrix}$$

same here  
after  $\frac{1}{2}$  hr.

(4) c. orthogonal projection of vectors in  $\mathbb{R}^2$  onto y axis.



(4) 16. Let  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^1$ . What values are possible for:

(a) nullity 2 3

(b) rank 1 0

ⓐ

(12) 15. Are the following functions linear? Show work.

(a)  $F((x_1, x_2)) = (x_1 + x_2, 2x_2 - 3x_1)$

Yes

$$F((x_1, x_2) + (y_1, y_2)) = (x_1 + x_2 + y_1 + y_2, 2(y_2 + x_2) - 3(x_1 + y_1))$$

$$= (x_1 + x_2 + y_1 + y_2, 2y_2 + 2x_2 - 3x_1 - 3y_1)$$

$$F(x_1, x_2) + F(y_1, y_2) = (x_1 + x_2, 2x_2 - 3x_1) + (y_1 + y_2, 2y_2 - 3y_1) = (x_1 + x_2 + y_1 + y_2, 2y_2 + 2x_2 - 3x_1 - 3y_1)$$

(b)  $F((x_1, x_2, x_3)) = (x_2 - x_1, x_2 + x_3 - 1)$

No.

$$F(k(x_1, x_2, x_3)) = (kx_2 - kx_1, kx_2 + kx_3 - 1)$$

$$kF(x_1, x_2, x_3) = k(x_2 - x_1, x_2 + x_3 - 1) = (kx_2 - kx_1, kx_2 + kx_3 - k)$$

(c)  $F\begin{pmatrix} a & b \\ c & d \end{pmatrix} = a + d - b + 2c$

$$Fh\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ka + kd - kb + 2kc = kF\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$F\begin{bmatrix} a & b \\ c & d \end{bmatrix} + F\begin{bmatrix} a' & b' \\ c' & d' \end{bmatrix} = a + d - b + 2c + a' + d' - b' + 2c'$$

$$F\left[\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} a' & b' \\ c' & d' \end{bmatrix}\right] = (a+a') + (d+d') - (b+b') + 2(c+c')$$

- (5) 16. State carefully the following definition: the vectors  $\underline{v}_1, \dots, \underline{v}_n$  are linearly independent if and only if

$$k_1 \underline{v}_1 + \dots + k_n \underline{v}_n = \underline{0} \text{ only if } k_1 = 0, \dots, k_n = 0$$

- (8) 17. Find a basis for the subspace of  $\mathbb{R}^4$  spanned by the vectors  $(1, 1, 2, 1)$ ,  $(2, -1, 5, 7)$ , and  $(-1, 3, 0, 9)$ .

$$\begin{bmatrix} 1 & -1 & 2 & 1 \\ 2 & -1 & 5 & 7 \\ -1 & 3 & 0 & 9 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & 1 & 1 & 5 \\ 0 & 2 & 2 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$(1, -1, 2, 1)$$

$$(0, 1, 1, 5)$$

$$(0, 0, 0, 0)$$

↑ ~~subspace~~ should be 0

- (8) 18. Show that the column vectors of an  $n \times n$  invertible matrix form a basis for  $\mathbb{R}^n$ .

invert  $\Rightarrow$  col vec are indep  
each other, basis of  $\mathbb{R}^n$

- (8) 19. Are the vectors  $(1, 2, 1)$ ,  $(2, 2, 3)$ , and  $(3, 8, -6)$  linearly independent?

$$\begin{vmatrix} 1 & 2 & 1 \\ 2 & 2 & 3 \\ 3 & 8 & -6 \end{vmatrix} \begin{vmatrix} 1 & 3 \\ 2 & 2 \\ 3 & 8 \end{vmatrix} = -12 + 4 + 16 - 6 - 24 + 24 = 0$$

$\neq 0$

Yes

assume  $\mathbb{R}^n$ ,  $M_{m \times n}$  are vector spaces.

(12) 20. Which of the following are subspaces of  $\mathbb{R}^3$ ? Justify.

(a) all vectors of form  $(a, a, 1)$

$$\text{NO } (a, a, 1) + (b, b, 1) = (a+b, a+b, 2)$$

(b) all vectors of form  $(0, 0, a)$

$$(0, 0, a) + (0, 0, b) = (0, 0, a+b)$$

$$k(0, 0, a) = (0, 0, ka)$$

(c) set of solutions of  $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

plane thru  $(0, 0, 0)$

(8) 21. Which of the following are vector spaces? Justify.

(a) set of all matrices of form  $\begin{bmatrix} a & b & 1 \\ 1 & a & b \end{bmatrix}$

$$\begin{bmatrix} a_1 & b_1 & 1 \\ 1 & a_1 & b_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 & 1 \\ 1 & a_2 & b_2 \end{bmatrix} = \begin{bmatrix} a_1+a_2 & b_1+b_2 & 2 \\ 2 & a_1+a_2 & b_1+b_2 \end{bmatrix}$$

NO

(b) set of all polynomials of the form  $ax + bx^2$ .

$$a_1x + b_1x^2 + a_2x + b_2x^2 = (a_1+a_2)x + (b_1+b_2)x^2$$

$$k(ax + bx^2) = kax + kbx^2$$

Yes.

(10) 22. Suppose the linear transformation  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  has matrix  $\begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}$ . Show work.

- (a) Is  $(4, -100)$  in the range of  $T$ ?
- (b) Is  $(4, -2)$  in the null space (kernel) of  $T$ ?
- (c) What is the rank of  $T$ ?
- (d) What is the nullity?

$$\begin{bmatrix} 2 & 5 \\ -1 & 10 \end{bmatrix}$$

*Ignore*

(12) 23. Suppose that the linear transformation  $T$  has matrix  $\begin{bmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & -2 & -1 & 3 \end{bmatrix}$ .

- 2 (a)  $T$  is from  $\mathbb{R}^4$  to  $\mathbb{R}^3$
- 4 (b) Find a basis for the null space.
- 4 (c) Find a basis for the range.
- 2 (d) What are the rank and nullity?

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & -2 & -1 & 3 \end{bmatrix}$$

b.  $\begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & -2 & -4 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

c.  $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 1 & 4 & -1 \\ 2 & 3 & 8 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & 2 & -4 \\ 0 & -1 & 2 \end{bmatrix}$

~~$(1, 0, 1, 2)$~~   
 ~~$(0, 1, 2, -1)$~~   
 $(-4, -2, 1, 0)$   
 $(-2, 1, 0, 1)$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

d. nullity = 2  
 rank = 2

$$\begin{bmatrix} (1, 2, 3) \\ (0, 1, -2) \end{bmatrix}$$

$$\begin{matrix} x_4 = t \\ x_3 = s \\ x_2 = -2s + t \\ x_1 = -s - 2t \end{matrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = s \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

Show work, give reasons.

(16) 1. If  $\vec{u} = (1, 2, -1, 0)$  and  $\vec{v} = (2, 2, 3, 6)$ , find:

a.  $\vec{u} + \vec{v} = (3, 4, 2, 6)$

b.  $3\vec{u} = (3, 6, -3, 0)$

c.  $\vec{u} - \vec{v} = (-1, 0, -4, -6)$

d.  $\vec{u} \cdot \vec{v} = 2 + 4 - 3 + 0 = 3$

e.  $7\vec{u} - \vec{v} = (7, 14, -7, 0) - (2, 2, 3, 6) = (5, 12, -10, -6)$

f.  $\|\vec{u}\| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$

g.  $\vec{u} \cdot (\vec{v} - 2\vec{u}) = (1, 2, -1, 0) \cdot [(2, 2, 3, 6) - (2, 4, -2, 0)]$   
 $= (1, 2, -1, 0) \cdot (0, -2, 5, 6)$   
 $= 0 - 4 - 5 + 0 = -9$

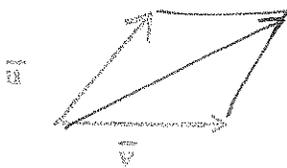
h. distance between the points  $\vec{u}$  and  $\vec{v}$  in  $\mathbb{R}^4$ .

$\sqrt{1^2 + 0 + 4^2 + 6^2} = \sqrt{54} = 3\sqrt{6}$

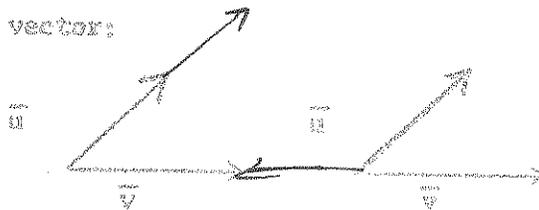
(3) 2. Are  $(1, 2, 3)$  and  $(3, 2, 0)$  perpendicular?

$(1, 2, 3) \cdot (3, 2, 0) = 3 + 4 = 7$  no

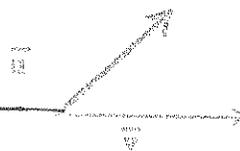
(8) 3. Draw the indicated vector:



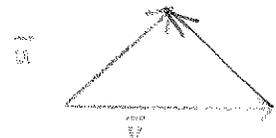
a.  $\vec{u} + \vec{v}$



b.  $2\vec{u}$



c.  $-\vec{v}$



d.  $\vec{u} - \vec{v}$

(3) 4.  $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 2 & 1 & 0 \end{bmatrix}^t = \begin{bmatrix} 1 & 4 & 2 \\ 2 & 5 & 1 \\ 3 & 6 & 0 \end{bmatrix}$

(3) 8. What is the cofactor  $C_{23}$  of  $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 0 & 7 \\ 7 & -8 & 10 \end{bmatrix}$ ?

$- \begin{vmatrix} 1 & -1 \\ 7 & -8 \end{vmatrix} = -(-8 + 7) = 1$

(12) 6. Find each determinant. Each can be done rather quickly without a lot of calculation. Give reasons for your answer or show calculation to get full credit.

$$a. \begin{vmatrix} 2 & 1 & 2 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 3 \end{vmatrix} = 2 \cdot 1 \cdot 1 \cdot (-3) = -6$$

$$b. \begin{vmatrix} 1 & 2 & 0 & 7 \\ 3 & 1 & 0 & 0 \\ 2 & 1 & 0 & 2 \\ 1 & 3 & 0 & 6 \end{vmatrix} = 0 \text{ row of zeros}$$

$$c. \begin{vmatrix} 1 & 2 & 2 & 1 \\ 2 & 4 & 1 & 1 \\ 2 & 4 & 1 & 2 \\ 3 & 6 & 3 & 3 \end{vmatrix} = 0 \text{ col 1 \& 2 prop.}$$

$$d. \begin{vmatrix} 0 & 2 & 1 & 1 \\ 1 & 0 & 3 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{vmatrix} = - \begin{vmatrix} 1 & 0 & 3 & 1 \\ 0 & 2 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{vmatrix} = -2$$

(18) 7. If  $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 \\ 1 & 1 & 2 & 3 \\ -1 & 1 & 2 & 7 \end{bmatrix}$ , and  $\det A = -8$ , find (each can be found from A):

$$a. \det A^T = -8$$

$$b. \det A^{-1} = \frac{1}{-8}$$

$$c. \det 2A = 2^4(-8) = -16 \cdot 8 = -128$$

$$a. \begin{vmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{vmatrix}$$

$$= 2 \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = -2 \cdot 1 = -2$$

$$d. \begin{vmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 \\ 1 & 1 & 2 & 3 \\ -1 & 1 & 2 & 7 \end{vmatrix} = -16 - 24$$

$$e. \begin{vmatrix} 1 & 1 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 3 \\ 1 & 1 & 2 & 7 \end{vmatrix} = 8$$

$$f. \begin{vmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 \\ 1 & 1 & 2 & 3 \\ 1 & 1 & 2 & 7 \end{vmatrix} = -8$$

$$\begin{aligned}
 (5) \quad & \begin{vmatrix} 2 & 0 & 1 & 0 \\ 1 & 1 & 1 & 2 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & 0 & 2 \end{vmatrix} = 2 \begin{vmatrix} 1 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{vmatrix} - \begin{vmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{vmatrix} \\
 & = -2 \cdot \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} - \left( - \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} \right) \\
 & = -2 \cdot 3 + 3 = -3
 \end{aligned}$$

$$(5) \quad \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}^{-1} = \frac{1}{2} \begin{bmatrix} 2 & -2 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

$$\det = 2$$

$$\text{adj} = \begin{bmatrix} \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 0 & 2 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} \\ -\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix}, -\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}, -\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \end{bmatrix}^t = \begin{bmatrix} 2 & 0 & 0 \\ -2 & 2 & 0 \\ 0 & -1 & 1 \end{bmatrix}^t = \begin{bmatrix} 2 & -2 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

(5) 10. Find the orthogonal projection of the vector  $u$   $(3, 6)$  onto the vector  $v$   $(-1, 2)$ . 1:00

$$\frac{u \cdot v}{v \cdot v} v = \frac{-3 + 12}{5} (-1, 2)$$

$$= \frac{9}{5} (-1, 2)$$

$$= \left( -\frac{9}{5}, \frac{18}{5} \right)$$

(5) 11. Is the matrix  $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 2 & 3 \\ 3 & 6 & -2 \end{bmatrix}$  invertible? Give reasons.

$$\det = \begin{vmatrix} 2 & 3 \\ 6 & -2 \end{vmatrix} + 3 \begin{vmatrix} 2 & -1 \\ 2 & 3 \end{vmatrix} = -4 - 18 + 6 + 2 = -20 \neq 0 \text{ Yes.}$$

(2) 12. Is the line  $x = 1 + t, y = 3 + t, z = 5, t \in \mathbb{R}$ , perpendicular to the plane  $x + y + 3z = 2$ ?

$$v = (1, 1, 0) \quad \bar{v} \cdot \bar{u} = 1 + 1 + 0 = 2 \neq 0$$

$$u = (1, 1, -3) \quad \bar{v} = k\bar{u} ? \quad \text{no!}$$

(5) 13. Find the equation of the plane containing the points  $(2, 1, 0), (3, 2, 6),$  and  $(1, 2, 2)$ .

$$(1, 1, 6) \times (-1, 1, 2)$$

$$\begin{vmatrix} 1 & 1 & 6 \\ -1 & 1 & 2 \end{vmatrix} = \left( \begin{vmatrix} 1 & 6 \\ 1 & 2 \end{vmatrix}, - \begin{vmatrix} 1 & 6 \\ -1 & 2 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} \right) = (-4, -8, 2)$$

$$(-4, -8, 2) \cdot (x-2, y-1, z) = 0$$

$$-4(x-2) - 8(y-1) + 2z = 0$$

(.) 14. Derive an expression for  $\det A^{-1}$ .

$$-4x - 8y + 2z + 16 = 0$$

$$AA^{-1} = I$$

$$\det A \cdot \det A^{-1} = \det I = 1$$

$$\det A^{-1} = \frac{1}{\det A}$$

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+ 4 free points

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