

MATH 332

Midterm Test

October 20, 1989

Name Kay

After 30° , 74°
down
clockwise
First left at 40°
 5 left by 50°
(10)

1. Complete the following DEFINITIONS:

a. The vectors v_1, v_2, \dots, v_n are linearly independent if and only if

$$k_1v_1 + \dots + k_nv_n = 0 \text{ and if } k_1 = 0, \dots, k_n = 0$$

Y

- b. A subset
- W
- of a vector space
- V
- is a
- subspace
- of
- V
- if and only if

W is also a vector space with the same
operations as V .

2. Complete
- and
- prove: (15)

a. For a fixed $m \times n$ matrix A , the set of all x such that $Ax = 0$ is a✓ subspace of \mathbb{R}^n .

$$Ax_1 = 0, Ax_2 = 0$$

$$A(x_1 + x_2) = Ax_1 + Ax_2 = 0 + 0 \text{ so closed under } +$$

$$A(kx_1) = kAx_1 = k \cdot 0 = 0$$

so multiply

- b. If
- A
- is invertible, then
- $\det A \neq 0$
- .

$$AA^{-1} = I$$

$$\det A \det A^{-1} = \det I = 1$$

$$\text{so } \det A \neq 0.$$

3. Write the elementary matrix which adds
- $2 \times$
- first row to the third row of a
- 3×3
- matrix. (5)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

4. Are the following vectors in \mathbb{R}^4 linearly independent? Justify. (10) *part*

$(1,0,0,1), (2,2,-2,2), (-1,1,1,-1)$.

$$k_1(1, 3, 4, 1) + k_2(2, 2, -2, 2) + k_3(-1, 1, 1, -1) = (0, 0, 0, 0)$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & -2 & 1 & 0 \\ 1 & 2 & -1 & 0 \end{array} \right] \rightarrow \left[\begin{array}{cccc|c} 1 & 2 & -1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \rightarrow \left[\begin{array}{cccc|c} 1 & 2 & -1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Yes only soln $k_1 = 0, k_2 = 0, k_3 = 0$

✓

5. Verify that the set of all polynomials of the form $ax^3 + ax + b$ is a subspace of P_3 . (10) *part*

$$(ax^3 + ax + b) + (a'x^3 + a'x + b')$$

$$= (a+a')x^3 + (a+a')x + (b+b') \text{ same form}$$

$$k(ax^3 + ax + b) = (ka)x^3 + (ka)x + kb \text{ so closed.}$$

3:20

✓

6. Find

$$\left[\begin{array}{cc} 2 & 9 \\ -2 & 3 \end{array} \right]^{-1} = \frac{1}{6+18} \left[\begin{array}{cc} 3 & -9 \\ 2 & 2 \end{array} \right] = \left[\begin{array}{cc} \frac{3}{24} & \frac{-9}{24} \\ \frac{2}{24} & \frac{2}{24} \end{array} \right] = \left[\begin{array}{cc} \frac{1}{8} & \frac{-3}{8} \\ \frac{1}{12} & \frac{1}{12} \end{array} \right]$$

out but not

7. Find a unit vector parallel to $(1, 2, 3)$. (5)

$$\|(1, 2, 3)\| = \sqrt{1+4+9} = \sqrt{14}$$

$$\frac{1}{\sqrt{14}} (1, 2, 3)$$

out but ✓

- 8.

$$\det \left[\begin{array}{cccc} -2 & 0 & 1 & 3 \\ 2 & 0 & 2 & 0 \\ 0 & 6 & -3 & 0 \\ 0 & 0 & 6 & -2 \end{array} \right] = -1 \det \left[\begin{array}{ccc} -2 & 1 & 3 \\ 2 & 2 & 0 \\ 0 & 6 & -2 \end{array} \right]$$

$$= - \det \left[\begin{array}{ccc} -2 & 1 & 3 \\ 0 & 3 & 3 \\ 0 & 6 & -2 \end{array} \right]$$

$$= - \det \left[\begin{array}{ccc} -2 & 1 & 3 \\ 0 & 3 & 3 \\ 0 & 0 & -8 \end{array} \right] = -(-2)(3)(-8)$$

$$= 48$$

(5)

more

out but ✓

9. Give an example of matrices A and B where $AB = 0$, but $A \neq 0$ and $B \neq 0$.

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (5)$$

all but 4

10. Complete the following operations (if not possible, say so.) (10)

a. $\begin{bmatrix} 1 & 3 & -3 \end{bmatrix} \cdot \begin{bmatrix} 2 & 8 & 0 \\ -2 & 5 & 1 \end{bmatrix} = X$

all but 2

b. $\begin{bmatrix} 1 & 3 & -3 \end{bmatrix} + \begin{bmatrix} 2 & 8 & 0 \\ -2 & 5 & 1 \end{bmatrix} = X$

11. Give all solutions for the following system of equations in matrix form.

$$\left[\begin{array}{ccccc|c} 2 & 0 & 8 & 4 & 2 & 0 \\ 0 & 0 & 2 & 8 & 4 & 0 \\ 0 & 0 & 0 & 1 & 5 & 0 \end{array} \right]$$

systm in vector for.

is good

$$x_5 = t \quad x_4 + 5t = 0 \quad x_4 = -5t$$

(10)

$$2x_3 + 8(-5t) + 4(t) = 0$$

$$x_1 = -126t - 63t$$

$$2x_3 = 36t \quad x_3 = 18t$$

$$x_2 = 5$$

$$2x_1 + 8(18t) + 4(-5t) + 2t = 0$$

$$x_4 = -18t$$

$$2x_1 + 144t - 20t + 2t = 0 \quad 2x_1 = -126t$$

$$x_5 = t$$

($\frac{1}{2}$ and $x_2 = 5$)

12. Express the matrix A as a linear combination of B and C if possible.

$$A = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$

(10)

$$k_1 \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix} + k_2 \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$$

$$k_1 + 2k_2 = 3$$

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 2 & 1 & 0 \\ 2 & 3 & 4 \\ 0 & 0 & 0 \end{array} \right] \rightarrow \left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -1 & -2 \\ 0 & 0 & 0 \end{array} \right]$$

$$2k_1 + k_2 = 0$$

$$-2k_1 = 0$$

$$2k_1 + 3k_2 = 4$$

$$2k_1 + 3k_2 = 4$$