

1. Complete the following DEFinitions: (10)

a. The matrix B is an inverse of the matrix A if and only if

$$AB = I \text{ and } BA = I \quad \text{all}$$

- b. A subset W of a vector space V is a subspace of V if and only if

$W$  is a vector space with the same ops  
as  $V$ .

2. Complete and prove: (15)

a. For a fixed  $m \times n$  matrix A, the set of all  $x$  such that  $Ax = 0$  is a subspace of  $\mathbb{R}^n$ .

$$(1) \quad \begin{aligned} Ax_1 &= 0 \\ Ax_2 &= 0 \end{aligned} \Rightarrow A(x_1 + x_2) = 0 \quad A(x_1 + x_2) = Ax_1 + Ax_2 = 0 + 0 = 0$$

$$(2) \quad Ax = 0 \Rightarrow A(kx) = kAx = k0 = 0$$

4/15

9 class

P

b. If A is invertible, then  $Ax = 0$  has only trivial soln

not very  
many.

3. Write the elementary matrix which adds  $3 \times$  second row to the third row of a  $3 \times 3$  matrix. (5)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

$\frac{20}{25}$

4. Find

$$\begin{bmatrix} 4 & 7 \\ -2 & 3 \end{bmatrix}^{-1} = \frac{1}{12+4} \begin{bmatrix} 3 & 7 \\ -2 & 4 \end{bmatrix} = \frac{1}{16} \begin{bmatrix} 3 & 7 \\ -2 & 4 \end{bmatrix}$$
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5. Find a vector of length one parallel to  $(-2, 2, 3)$ .

$$\frac{1}{\sqrt{(-2)^2 + 2^2 + 3^2}} (-2, 2, 3) = \frac{1}{\sqrt{17}} (-2, 2, 3)$$
all

6.

$$\begin{aligned} \det \begin{bmatrix} -3 & 0 & 1 & 3 \\ 3 & 0 & 2 & 0 \\ 0 & 1 & 6 & -3 \\ 0 & 0 & 6 & -2 \end{bmatrix} &= -\det \begin{bmatrix} -3 & 1 & 3 \\ 3 & 2 & 0 \\ 0 & 6 & -2 \end{bmatrix} = -\det \begin{bmatrix} -3 & 1 & 3 \\ 0 & 3 & 3 \\ 0 & 6 & -2 \end{bmatrix} \quad \text{19/23} \\ &= -(-3 \det \begin{bmatrix} 2 & 0 \\ 6 & -2 \end{bmatrix}) = -\det \begin{bmatrix} -3 & 3 & 3 \\ 6 & -2 \end{bmatrix} \\ &= -(-3 \begin{vmatrix} 2 & 0 \\ 6 & -2 \end{vmatrix} + 3 \begin{vmatrix} 1 & 3 \\ 6 & -2 \end{vmatrix}) = 3 \begin{bmatrix} -6 & -18 \end{bmatrix} = -72 \end{aligned}$$

7. Do the following vectors span all of  $\mathbb{R}^4$ ? Justify. $(1, 0, 0, 1), (0, 2, -2, 2), (0, 0, 1, -1), (0, 0, -2, 2)$ 

$$k_1(1, 0, 0, 1) + k_2(0, 2, -2, 2) + k_3(0, 0, 1, -1) + k_4(0, 0, -2, 2) = (b_1, b_2, b_3, b_4)$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & -2 & 1 & -2 \\ 1 & 2 & -1 & 2 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} = b$$
V2

note for all  $b$ ?

$$\det \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & -2 & 1 & -2 \\ 1 & 2 & -1 & 2 \end{bmatrix} = \det \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 2 & -1 & 2 \end{bmatrix} = 0 \text{ ms.}$$

8. Find the equation of the plane through the point  $(1, 0, 3)$  which is perpendicular to the planes  $x - y = 5$  and  $2x + 3y - z = 4$ .

$$\vec{n}_1 = (1, -1, 0) \quad \vec{n}_2 = (2, 3, -1)$$

$$\vec{n} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} i & j & k \\ 1 & -1 & 0 \\ 2 & 3 & -1 \end{vmatrix} = (-1, 1, 5) = \vec{i} - \vec{j} + 5\vec{k}$$

$$= \vec{x} + \vec{y} + 5\vec{z}$$
 $\frac{1}{2}$

$$(1, 1, 5)(x-1, y, z-3) = 0$$

$$x-1 + y + 5(z-3) = 0$$

$$x+y+5z = 16$$

9. For the following matrix, find all solutions of the system of equations  $\mathbf{A}\mathbf{x} = \mathbf{0}$ . Write in the form of a linear combination of vectors. (10)

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 0 & 3 \\ -2 & -2 & 1 & -5 \\ -1 & 4 & 3 & 0 \end{bmatrix}$$

↑ confused form

none

8 others

done  
partial  
incorrect  
good

$$\begin{bmatrix} 1 & 2 & 0 & 3 \\ -2 & -2 & 1 & -5 \\ -1 & 4 & 3 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 2 & 1 & 1 \\ 0 & 6 & 3 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_1 = t \quad x_3 = s$$

$$2x_2 + t + s = 0$$

$$x_2 = -\frac{1}{2}t - \frac{1}{2}s$$

$$x_1 = -2t + s$$

$$x_2 = -\frac{1}{2}t - \frac{1}{2}s$$

$$x_3 = s$$

$$x_4 = t$$

$$x_1 + 2x_2 + 3t = 0$$

$$(x_1, x_2, x_3, x_4) = s(1, -\frac{1}{2}, 1, 0) + t(-2, -\frac{1}{2}, 1, 1)$$

$$x_1 + (-t-s) + 3t = 0 \quad x_1 = -2t + s$$

10. Find the distance between the parallel planes  $x - 2y + 4z = 3$ , and (8)

$$x - 2y + 4z = 7.$$

$$\text{Pt } (1, 1, 1) \text{ on } (3, 0, 0) \quad \text{Pt } (7, 0, 0)$$

$$\mathbf{n} = (1, -2, 4)$$

$$\bar{a} = (4, 0, 0)$$

$\frac{11}{25}$

some calc

proj  $\bar{a}$  onto  $\mathbf{n}$

$$\bar{p} = \frac{(4, 0, 0) \cdot (1, -2, 4)}{(1, -2, 4) \cdot (1, -2, 4)} (1, -2, 4) = \frac{4}{21} (1, -2, 4)$$

$$\|\bar{p}\| = \frac{4}{\sqrt{21}}$$

11. Express the polynomial  $r(x) = -4 + 4x - 36x^2 + 4x^3$  as a linear combination of  $p(x) = 1 + 2x - x^3$  and  $q(x) = 2 + 12x^2 - 2x^3$ , if possible. (10)

$$k_1(1, 2, -1)$$

make not possible

$$k_1(1, 2, -1) + k_2(2, 12, -2) = -4 + 4x - 36x^2 + 4x^3$$

$$-k_1 + 2k_2 = -4$$

$$k_1 + 2k_2 = -4$$

$$2k_1 = 4$$

$$12k_2 = -36$$

$$\left[ \begin{array}{cc|c} 1 & 2 & -4 \\ 2 & 0 & 4 \\ 0 & 12 & -36 \\ -1 & -2 & 4 \end{array} \right] \rightarrow \left[ \begin{array}{cc|c} 1 & 2 & -4 \\ 0 & -4 & 12 \\ 0 & 12 & -36 \\ 0 & 0 & 0 \end{array} \right]$$

$$-k_1 - 2k_2 = 4$$

$$\left[ \begin{array}{cc|c} 1 & 2 & -4 \\ 0 & -4 & 12 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right] \quad k_2 = 3 \quad \frac{9}{25}$$

$$k_1 = -6 = 4 \quad k_1 = 2$$

$$r(x) = 2p(x) - 3q(x)$$

12. Verify that the set of all  $2 \times 2$  matrices of the following form, is a subspace of  $M_{22}$ .

$\begin{bmatrix} a & 0 \\ a & b \end{bmatrix}$ , where a and b are real numbers. (10)

1)  $\begin{bmatrix} a & 0 \\ a & b \end{bmatrix} + \begin{bmatrix} c & 0 \\ c & d \end{bmatrix} = \begin{bmatrix} a+c & 0 \\ a+c & b+d \end{bmatrix}$  same for

2)  $k \begin{bmatrix} a & 0 \\ a & b \end{bmatrix} = \begin{bmatrix} ka & 0 \\ ka & kb \end{bmatrix}$  same for

9/25

very  
close

Second do  
example only