

MATH 332

Midterm Test

October 25, 1993

Name _____

Key

Time = everyone = 1 hour

First left is 37

then a boat in 40°

cheating

X = 746

(25)

m = 72

I. Give complete DEFINITIONS for each of the following:

a. A matrix A is symmetric

b. The rank of a matrix

c. The set of vectors v_1, v_2, \dots, v_n is a basis for the subspace W

d. The null space of a matrix A

e. The dimension of a subspace W

II. Prove: If the dimension of W is ~~p~~, then any set of $p+1$ or more vectors is linearly dependent. (10)

wrong one

Omitted

III. Problems: (Show work!) Fill in the box when you see R.

1. Find all solutions for the system of linear equations (10)

$$\begin{aligned}x_1 + 2x_2 + 3x_3 - x_4 &= 2 \\x_2 - x_3 &= 5\end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & -1 & 2 \\ 0 & 1 & -1 & 0 & 5 \end{array} \right]$$

$$x_4 = t \quad x_3 = s$$

$$x_1 = -8 + 5s + t$$

$$x_2 - s = 5 \quad x_2 = 5 + s$$

$$+1s + 2s + 3s - t \\ x_1 + 2(5+s) + 3s - t = 2$$

$$\left[\begin{array}{c} -5s+t-8 \\ 5+s \\ s \\ t \end{array} \right]$$

2. Show that the set of vectors (x_1, x_2, x_3) for which $x_1 + x_2 + x_3 = 1$, is NOT a subspace of \mathbb{R}^3 . (5)

$$(0, 0, 0) \text{ not in set} \quad 0+0+0 \neq 1$$

14/20

3. a. The vectors $(1, 1, 1)$ and $(0, 1, 1)$ cannot form a basis for \mathbb{R}^3 . How do we know that? 7
 b. Do the vectors $(1, 1, 1)$, $(0, 1, 1)$ and $(0, 0, 1)$ form a basis for \mathbb{R}^3 ? Justify. (10)

(a) Because a basis for \mathbb{R}^3 must contain 3 vectors

$$(b) \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \text{ by row op } \Rightarrow \text{basis}$$

$$\text{or } \left[\begin{array}{ccc} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{array} \right] \text{ dim (Param space)} = 3$$

4. Find the inverse of the matrix

$$\left[\begin{array}{ccc} 2 & 2 & 3 \\ -6 & -7 & -7 \\ 2 & 0 & 8 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 2 & 2 & 3 & 1 & 0 & 0 \\ -6 & -7 & -7 & 0 & 1 & 0 \\ 2 & 0 & 8 & 0 & 0 & 1 \end{array} \right]$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 2 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & 2 & 3 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right]$$

(10)

10

$$\left[\begin{array}{ccc|ccc} 2 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & 2 & 3 & 1 & 0 \\ 0 & 0 & 1 & -7 & -2 & 1 \end{array} \right] \rightarrow \left[\begin{array}{ccc|ccc} 2 & 2 & 0 & 1 & 2 & 0 \\ 0 & -1 & 0 & 17 & 5 & -2 \\ 0 & 0 & 1 & -7 & -2 & 1 \end{array} \right] \rightarrow \left[\begin{array}{ccc|ccc} 2 & 0 & 0 & 5 & 16 & -7 \\ 0 & -1 & 0 & 17 & 5 & -2 \\ 0 & 0 & 1 & -7 & -2 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc} 28 & 7 & -3 \\ -17 & -5 & 2 \\ -7 & -2 & 1 \end{array} \right] \quad \left[\begin{array}{ccc} 28 & 8 & -7/2 \\ -17 & -5 & 2 \\ -7 & -2 & 1 \end{array} \right]$$

5. Find a basis for the subspace spanned by the vectors $(2, 1, 3, -1)$, $(2, 2, 5, -1)$, and $(-4, -1, -4, 2)$. This is a 2 dimensional subspace of \mathbb{R}^4 (10)

$$\left[\begin{array}{cccc} 2 & 1 & 3 & -1 \\ 2 & 2 & 5 & -1 \\ -4 & -1 & -4 & 2 \end{array} \right] \xrightarrow{\textcircled{1}} \left[\begin{array}{cccc} 2 & 1 & 3 & -1 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{array} \right] \xrightarrow{\textcircled{2}} \left[\begin{array}{cccc} 2 & 1 & 3 & -1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$(2, 1, 3, -1) \quad (0, 1, 2, 0)$$

6. Find a basis for the null space of the following matrix. may miss (10)

This is a 2 dimensional subspace of \mathbb{R}^4

$$\left[\begin{array}{cccc} 1 & 2 & 0 & 3 \\ 1 & 2 & 2 & 6 \\ -2 & -4 & 2 & -3 \end{array} \right] \rightarrow \left[\begin{array}{cccc} 1 & 2 & 0 & 3 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 2 & 3 \end{array} \right] \rightarrow \left[\begin{array}{cccc} 1 & 2 & 0 & 3 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

2 all
2 close

$$x_4 = t \quad 2x_3 + 3t = 0 \quad x_3 = -\frac{3}{2}t$$

$$x_2 = s \quad x_1 + 2s + 3t = 0 \quad x_1 = -2s - 3t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = s \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} -3 \\ 0 \\ \frac{3}{2} \\ 1 \end{bmatrix}$$

$$(-2, 1, 0, 0)$$

$$(-3, 0, \frac{3}{2}, 1)$$

as a basis

7. Show that the function F from \mathbb{R}^2 to \mathbb{R}^2 is a linear transformation: (10)

$$F\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = x_1 + x_2 \quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 \end{bmatrix}$$

few

$$F\left(\begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 \end{bmatrix}\right) = x_1 + x_2 + y_1 + y_2$$

$$F\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + F\left(\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}\right)\right) = x_1 + x_2 + y_1 + y_2$$

$$F\left(r\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = F\left(\begin{bmatrix} rx_1 \\ rx_2 \end{bmatrix}\right) = rx_1 + rx_2$$

$$rF\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = r(x_1 + x_2) = rx_1 + rx_2$$