

Name Key

- I. Give complete definitions for each (20)
- The set of vectors v_1, v_2, \dots, v_n is linearly independent

15

- The span of (or the subset of \mathbb{R}^n spanned by) the set of vectors v_1, v_2, \dots, v_n is

16

- A function $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$ is called onto

15

- A function $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$ is a linear transformation

14

- II. Prove (and complete where blank): (14)

- If the set of vectors v_1, v_2, \dots, v_n is linearly dependent, then at least one can be written as a linear combination of the others.

1
2

8
6

- If A is an invertible matrix, then $\det(A^{-1}) = \underline{\hspace{2cm}}$

3

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III. Problems (justify answers):

1. Compute (if not possible, say so):

(23)

a. $\begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}^{-1} = \frac{1}{-1-6} \begin{bmatrix} 1 & -2 \\ -3 & -1 \end{bmatrix} = -\frac{1}{7} \begin{bmatrix} 1 & -2 \\ -3 & -1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{7} & \frac{2}{7} \\ \frac{3}{7} & \frac{1}{7} \end{bmatrix}$ all but 2

b. $3 \begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} + 2 \begin{bmatrix} 2 & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ 0 & 15 \end{bmatrix} + \begin{bmatrix} 4 & -4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 7 & -7 \\ 0 & 17 \end{bmatrix}$ all but 2

c. $\begin{bmatrix} 1 & -2 & 1 \\ 0 & 5 & 2 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ 2 & 1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 10 & 15 \end{bmatrix}$ all but 6

d. $\begin{bmatrix} 1 & -2 \\ 4 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 & 3 \\ 4 & 5 & -6 \end{bmatrix} = \text{X}$ all

e. $\begin{bmatrix} -11 & -2 & 1 \\ 0 & 5 & 2 \end{bmatrix}^T = \begin{bmatrix} -11 & 0 \\ -2 & 5 \\ 1 & 2 \end{bmatrix}$ all but 2

f. $\det \begin{bmatrix} 2 & 3 & 5 \\ 0 & -1 & 2 \\ 0 & 0 & 5 \end{bmatrix} = 2(-1)(5) = -10$ all but 1
many did by row

g. $\det \begin{bmatrix} 2 & 3 & 5 & 10 \\ 0 & -1 & 2 & 3 \\ 0 & 0 & 0 & 5 \\ 4 & 2 & 2 & 2 \end{bmatrix} = -5 \det \begin{bmatrix} 2 & 3 & 5 \\ 0 & -1 & 2 \\ 4 & 2 & 2 \end{bmatrix} = (-5) [2 \left| \begin{smallmatrix} -1 & 2 \\ 2 & 2 \end{smallmatrix} \right| + 4 \left| \begin{smallmatrix} 3 & 0 \\ -1 & 2 \end{smallmatrix} \right|] = -5 [2(-6) + 4(11)] = (-5)(-12+44) = (-5)(32) = -160$

2. find the inverse of

$$\begin{bmatrix} 1 & 3 & 4 \\ -2 & -3 & -6 \\ 4 & 3 & 8 \end{bmatrix} \left| \begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ 0 & -1 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right. \rightarrow \left| \begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ 0 & 3 & 2 & 0 & 1 & 0 \\ 0 & -9 & -8 & -4 & 0 & 1 \end{array} \right| \rightarrow \left| \begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ 0 & 3 & 2 & 0 & 1 & 0 \\ 0 & 0 & -2 & 2 & 3 & 1 \end{array} \right|$$

all but (7) 16

$$\left| \begin{array}{ccc|ccc} 1 & 3 & 0 & -3 & -6 & -2 \\ 0 & 3 & 0 & 0 & -2 & -1 \\ 0 & 0 & 2 & 2 & 3 & 1 \end{array} \right| \rightarrow \left| \begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & -4 & -1 \\ 0 & 3 & 0 & 0 & -2 & -1 \\ 0 & 0 & 2 & 2 & 3 & 1 \end{array} \right| \rightarrow \left| \begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & -4 & -1 \\ 0 & 1 & 0 & 0 & -2 & -1 \\ 0 & 0 & 1 & 1 & 3 & 1 \end{array} \right|$$

$$\left| \begin{array}{ccc|ccc} 1 & 3 & 0 & -5 & -6 & -2 \\ 0 & 3 & 0 & 4 & 4 & 1 \\ 0 & 0 & -2 & 2 & 3 & 1 \end{array} \right| \rightarrow \left| \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & -1 \\ 0 & 3 & 0 & 4 & 4 & 1 \\ 0 & 0 & -2 & 2 & 3 & 1 \end{array} \right| \rightarrow \left| \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & -1 \\ 0 & 1 & 0 & 4/3 & 4/3 & 1/3 \\ 0 & 0 & 1 & -1 & -3/2 & -1/2 \end{array} \right|$$

$$\left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$$

2

20 min

3. Find the matrix for the linear transformation $T(x_1, x_2, x_3) = (x_1 - 2x_2, 3x_1)$. (3)

$$T(1, 0, 0) = (1, 3)$$

$$T(0, 1, 0) = (-3, 0)$$

$$T(0, 0, 1) = (0, 0)$$

$$\begin{bmatrix} 1 & -2 & 0 \\ 3 & 0 & 0 \end{bmatrix}$$

$$114^{\circ} 80'$$

8

720°

4. Find all solutions for the system of equations $x_1 + 2x_2 - x_4 = 9$, $x_1 + x_3 + x_4 = 6$.

Write as a linear combination of vectors for

Sol

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & -1 & 9 \\ 1 & 0 & 1 & 1 & 6 \end{array} \right] \rightarrow \left[\begin{array}{cccc|c} 1 & 2 & 0 & -1 & 9 \\ 0 & -2 & 1 & 2 & -3 \end{array} \right]$$

$$x_4 = t \quad x_3 = s \quad -2x_2 + s + 2t = -3$$

$$x_2 = \frac{1}{2}s + t + \frac{3}{2}$$

$$x_1 + 2\left(\frac{1}{2}s + t + \frac{3}{2}\right) - t = 9$$

$$x_1 + s + 2t + 3 - t = 9$$

$$x_1 = -s - t + 6$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = t \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} + s \begin{bmatrix} -1 \\ \frac{1}{2} \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 6 \\ \frac{3}{2} \\ 0 \\ 0 \end{bmatrix}$$

(6)

8
17 done

5. Is the vector $(2, 2, 8, 5)$ in the span of the vectors $(1, 2, 0, -1)$, $(2, 6, -6, -2)$, and $(-3, -5, -1, 7)$?

These vectors are in \mathbb{R}^4 .

(8)

$$x_1 \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 6 \\ -6 \\ -3 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ -5 \\ -1 \\ 7 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 8 \\ 5 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -3 & 2 \\ 2 & 6 & -5 & 2 \\ 0 & -6 & -1 & 8 \\ -1 & -2 & 7 & 5 \end{array} \right] \rightarrow \left[\begin{array}{cccc|c} 1 & 2 & -3 & 2 \\ 0 & 2 & 1 & -2 \\ 0 & -6 & -1 & 8 \\ 0 & 0 & 4 & 7 \end{array} \right] \rightarrow \left[\begin{array}{cccc|c} 1 & 2 & -3 & 2 \\ 0 & 2 & 1 & -2 \\ 0 & 0 & 4 & 7 \\ 0 & 0 & 4 & 7 \end{array} \right] \rightarrow \left[\begin{array}{cccc|c} 1 & 2 & -3 & 2 \\ 0 & 2 & 1 & -2 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 \end{array} \right]$$

no sols.

not in span

V7

6. Are the vectors $(1, -2, 0, 1, 0)$, $(0, 2, 6, 0, 4)$, and $(2, -2, 5, 2, 6)$ linearly independent? (8)

$$x_1 \begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 2 \\ 6 \\ 0 \\ 4 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ -2 \\ 5 \\ 0 \\ 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 2 & 0 & 0 \\ -2 & 2 & -2 & 0 & 0 \\ 0 & 6 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 6 & 0 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccccc|c} 1 & 0 & 2 & 0 & 0 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 6 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 6 & 0 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccccc|c} 1 & 0 & 2 & 0 & 0 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccccc|c} 1 & 0 & 2 & 0 & 0 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

11
most show
Wrong cond.
Some rows

only $(0, 0, 0)$ sol, Yes

yes

7. Do the columns of the following matrix span \mathbb{R}^3 ?

(4)

$$Q \begin{bmatrix} 2 & 1 & 0 \\ 0 & -5 & -6 \\ 0 & 0 & 7 \end{bmatrix}$$

$$\det = -20 \neq 0 \quad \text{yes}$$

8. The linear transformation T is given by $T(\underline{x}) = A \underline{x}$ where

(7)

$$A = \begin{bmatrix} 2 & -1 \\ 2 & 1 \\ 0 & 2 \end{bmatrix}$$

1. a. This is a linear transformation from \mathbb{R}^2 to \mathbb{R}^3 .
2. b. Is this transformation 1-1? (justify)
3. c. The range of T is the span of what vectors?

$$\begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \quad \text{anyone rel?}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & -2 \\ 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 1 \\ 0 & -2 \\ 0 & 0 \end{bmatrix} \quad \text{yes}$$

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