

$4\frac{1}{2}$ hrs 15 min

First left after
 $\frac{1}{2}$ hour

$$\bar{x} = 86.6$$

MATH 322

Test I

April 21, 1981

Name **KET**

- (27) 1. Perform the following calculations. If not possible, say so.

$$P_1: \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

$$P_2: \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$P_3: \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 12 & 18 \\ 18 & 12 & -6 \end{bmatrix}$$

$$P_4: \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} = X$$

$$P_5: \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} 13 & 24 \\ 24 & 45 \end{bmatrix}$$

$$P_6: \text{det } A = 21 \cdot 6 - 5 = 121 \quad P_7: \frac{1}{5} \begin{bmatrix} 7 & -4 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} \frac{7}{5} & -\frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{bmatrix}$$

$$P_8: \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} = X$$

$$P_9: \begin{bmatrix} 2 & 3 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 13 & -1 \\ 26 & -2 \end{bmatrix}$$

$$P_{10}: \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 0 \end{bmatrix} = -1 + 8 + 0 - (-3) - 0 - 4 \\ 7 + 3 - 4 = 6$$

(6) 2. Give the augmented matrix for each of the following systems of equations.

a. $x_1 + x_2 = 2$

$$\begin{array}{l} x_2 - x_3 = 3 \\ x_1 - x_2 - x_3 = -3 \end{array}$$

$$\left[\begin{array}{ccc|c} 0 & 1 & 1 & 5 \\ 1 & -1 & -1 & -1 \end{array} \right]$$

b. $x_1 + x_2 = 2$

$$\begin{array}{l} x_1 - x_3 = 3 \\ x_2 + x_3 = 7 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 1 & 0 & -1 & 3 \\ 0 & 1 & 1 & 7 \end{array} \right]$$

c. $x_1 + x_2 = 2$

$$\begin{array}{l} x_2 + x_3 = 7 \\ x_2 + x_3 + 5x_4 = 0 \\ x_1 - x_2 + x_3 = 2 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & 1 & 7 \\ 0 & 1 & 1 & 5 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 & 2 \end{array} \right]$$

(7) 3. Systems of equations have been reduced to the following. Find all solutions of each.

a. $\left[\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 3 \\ 0 & 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & 0 & 0 \end{array} \right]$

$$\begin{array}{l} x_3 = 0 \quad x_2 + 0 = 2 \quad x_2 = 2 \\ x_1 - 2 = 3 \quad x_1 = 5 \\ x_1 + 4 = 3 \end{array} \quad (5, 2, 0)$$

b. $\left[\begin{array}{cccc|c} 1 & 0 & 2 & 3 & 1 \\ 0 & 1 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$

no sol.

c. $\left[\begin{array}{cccc|c} 1 & 2 & 3 & 6 & 1 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{array} \right]$

$$\begin{array}{l} x_3 = t \quad x_1 + 2(2) + 3t = 6 \\ x_2 = 2 \quad x_1 = 2 - 3t \\ (2 - 3t, 2, t) \end{array}$$

d. $\left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 5 \\ 0 & 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right]$

$$\begin{array}{l} x_4 = 0 \quad x_2 = t \\ x_3 = 2 \quad x_1 - t + 4 = 6 \\ x_1 = 2 + t \\ (2 + t, t, 2, 0) \end{array}$$

(i)

4. Solve the following system by Gaussian elimination.

$$2x_1 + 3x_2 - x_3 = 0$$

$$x_1 + x_2 + x_3 = 1$$

$$3x_1 + 5x_2 - 3x_3 = -1$$

$$\left[\begin{array}{ccc|c} 2 & 3 & -1 & 0 \\ 1 & 1 & 1 & 1 \\ 3 & 5 & -3 & -1 \end{array} \right]$$

$$\leftrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & 3 & -1 & 0 \\ 3 & 5 & -3 & -1 \end{array} \right] \begin{matrix} \textcircled{1}-2\textcircled{2} \\ \textcircled{3}-3\textcircled{1} \end{matrix} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -3 & -2 \\ 0 & 2 & -6 & -4 \end{array} \right]$$

$$\textcircled{3}-2\textcircled{2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -3 & -2 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$x_3 = t$$

$$x_2 - 3t = -2$$

$$x_2 = -2 + 3t$$

$$x_1 + (-2 + 3t) + t = 1$$

$$x_1 = 1 + 2 - 4t$$

$$= 3 - 4t$$

$$(3 - 4t, -2 + 3t, t)$$

(10) 5. Find the inverse of $\begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 1 \\ 1 & 1 & 3 \end{bmatrix}$. Check.

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 0 & 0 \\ 4 & -6 & 1 & 0 & 1 & 0 \\ 1 & 1 & 3 & 0 & 0 & 1 \end{array} \right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 0 & 0 & 1 \\ 4 & -6 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{array} \right]$$

$$\textcircled{2}-4\textcircled{1} \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 0 & 0 & 1 \\ 0 & -10 & -4 & 0 & 1 & -4 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & -4 \\ 0 & -10 & -4 & 0 & 1 & -4 \end{array} \right]$$

$$\textcircled{3} + 10\textcircled{2} \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 10 & 1 & -4 \end{array} \right] - \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -10 & -1 & 4 \end{array} \right]$$

$$\textcircled{1}-3\textcircled{3} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & +3 & 0 & 3 & -1 \\ 0 & 1 & 0 & 11 & 1 & -4 \\ 0 & 0 & 1 & -10 & -1 & 4 \end{array} \right]$$

$$\textcircled{1}-\textcircled{2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 19 & 2 & -7 \\ 0 & 1 & 0 & 11 & 1 & -4 \\ 0 & 0 & 1 & -10 & -1 & 4 \end{array} \right]$$

$$J_{inv} = \begin{bmatrix} 19 & 2 & -7 \\ 11 & 1 & -4 \\ -10 & -1 & 4 \end{bmatrix}$$

ok

$$\left[\begin{array}{ccc} 0 & 1 & 1 \\ 4 & -6 & 1 \\ 1 & 1 & 3 \end{array} \right] \left[\begin{array}{ccc} 19 & 2 & -7 \\ 11 & 1 & -4 \\ -10 & -1 & 4 \end{array} \right]$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- (5) 6. Is the following matrix invertible? (Hint: Don't calculate it. Notice that (1,3,2,2) is a solution of a certain system of equations.) Justify.

$$\begin{bmatrix} 2 & 2 & -3 & -1 \\ 1 & 1 & -1 & -2 \\ 0 & 2 & 0 & -3 \\ -3 & 1 & 0 & -1 \end{bmatrix}$$

$$2x_1 + 2x_2 - 3x_3 - x_4 = 0$$

$$2 + 6 - 6 - 2 = 0$$

$$x_1 + x_2 - x_3 - x_4 = 0$$

$$1 + 3 - 2 - 2 = 0$$

$$2x_2 - 3x_4 = 0$$

$$6 - 6 = 0$$

$$-x_1 + x_2 - x_4 = 0$$

$$-1 + 3 - 2 = 0$$

has natural soln.
so not
invertible

- (6) 7. Give elementary matrices for each of the following row operations for a 4x4 matrix.

a. Interchange rows 2 and 4.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 \end{bmatrix}$$

b. Add 3 times row 1 to row 4.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c. Multiply row 3 by 6.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 3 \end{bmatrix}$$

yes

- (5) 8. Find a matrix A such that AB will be the matrix resulting from multiplying the last row of B by 3 and adding the first row to it. Does it matter which order these are done in? B is 4x4 matrix.

- (2) 9. Why must the matrix A be square in order to define A^2 ?

not same size

- (2) 10. Find an expression for $(A + B)^2$ valid for all square A,B of the same size. Use matrix algebra.

$$(A+B)^2 = A^2 + AB + BA + B^2$$

(5) 11. Show that if B and C are both inverses for A, then B = C.

$$AB = I$$

$$C(AB) = CI = C$$

$$(CA)B = C$$

$$I B = C$$

$$B = C$$

(10) 12. Show: $(kA)^n = k^n A^n$, for all integers n.

$$n > 0 \quad (kA)^n = kA \cdots kA = k^n A^n$$

$$\begin{aligned} n < 0 \\ n = -m \\ m > 0 \end{aligned} \quad \left| \begin{aligned} (kA)^{-m} &= (kA)^{-1} (kA)^{-1} \cdots (kA)^{-1} \\ &= \frac{1}{k} A^{-1} \frac{1}{k} A^{-1} \cdots \frac{1}{k} A^{-1} \\ &= \frac{1}{k^m} A^{-m} = k^{-m} A^{-m} \\ &= k^n A^n \end{aligned} \right.$$

$$\overbrace{\quad\quad\quad}^{\left((kA)^{-1}\right)^m}$$

$$\left(\frac{1}{k} A^{-1}\right)^m$$

$$\left(\frac{1}{k}\right)^m (A^{-1})^m \text{ part a}$$

$$k^{-m} A^{-m}$$

$$k^n A^n$$