\tbh 332
st I
uary 23, 1976 Name KEY

work!

1. If A = (2,1,2), and B = (1,-1,3), find:

$\|B\|^2 = 1+1+9$

a. A + B = (3, 0, 5)

b. A + 3B = (2, 1, 2) + (3, -3, 9) = (5, -2, 11)

c. 3A = (6, 3, 6)

d. B - A = (-1, -2, 1)

e. A·B = 2 - 1 + 6 = 7

f. P the projection of A on B

$\frac{A\cdot B}{\|B\|^2}B = \frac{7}{11}(1,-1,3) = \left(\frac{7}{11}, -\frac{7}{11}, \frac{21}{11}\right)$

g. $\|A\| = \sqrt{4+1+4} = \sqrt{9} = 3$

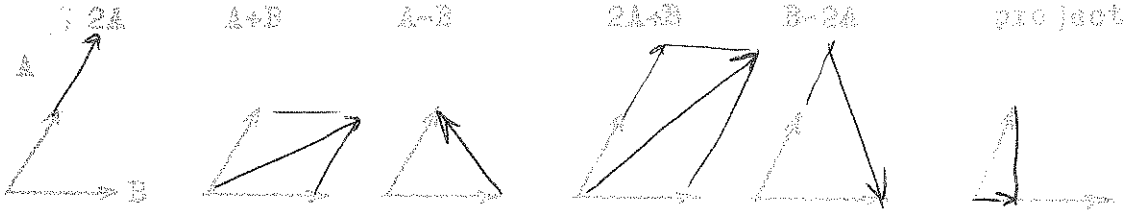
h. a unit vector orthogonal to A

$2\cdot 2 + 1\cdot 2 - 2\cdot 1 = 4 - 2 - 2 = 0$

$(2, -2, -1)$ $\left(\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}\right)$ or $\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$

10 2. In the following diagram carefully draw:

2A A+B A-B 2A+B B-2A projection of A on B



15 3. If $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$ $D = \begin{bmatrix} -1 & -2 \end{bmatrix}$, compute (if possible):

a. $A + B = \begin{bmatrix} 0 & 2 & 3 \\ 3 & 2 & 4 \end{bmatrix}$

b. A + C = X

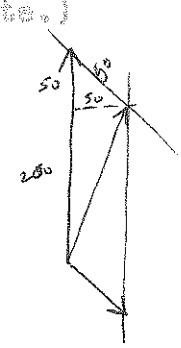
c. $A' = \begin{bmatrix} 1 & 3 \\ 2 & 1 \\ 1 & 2 \end{bmatrix}$

d. $DA = \underset{2\times 2}{\begin{bmatrix} -1, -2 \end{bmatrix}} \underset{2\times 3}{\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}} = \begin{bmatrix} -7, -4, -5 \end{bmatrix}$

e. $3B = \begin{bmatrix} -3 & 0 & 6 \\ 0 & 3 & 6 \end{bmatrix}$

f. AC = [X]

10 4. If plane is headed due north at 500 mph and there is a 50 mph wind out of the northwest, what is the ground velocity? (Draw a diagram and estimate.)



$\cancel{50\sqrt{1+4^2} = 50\sqrt{17}}$ $\approx 200+$

$\tan = \frac{1}{4}$

55

ath 232
est I
uary 23, 1976 Name KEY

work!

$2\frac{1}{2}$ pts

1. If $A = (2,1,2)$, and $B = (1,-1,3)$, find:

$\|B\|^2 = 1+1+9$

a. $A + B = (3, 0, 5)$

b. $A + 3B = (2,1,2) + (3,-3,9) = (5,-2,11)$

c. $3A = (6, 3, 6)$

d. $B - A = (-1, -2, 1)$

e. $A \cdot B = 2 - 1 + 6 = 7$

f. P the projection of A on B

$\frac{A \cdot B}{\|B\|^2} B = \frac{7}{11}(1,-1,3) = \left(\frac{7}{11}, -\frac{7}{11}, \frac{21}{11}\right)$

g. $\|A\| = \sqrt{4+1+4} = \sqrt{9} = 3$

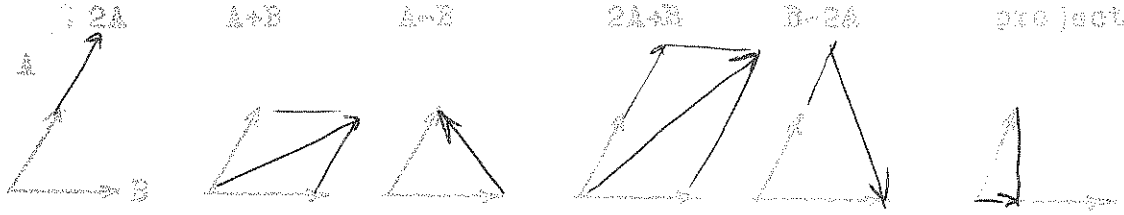
h. a unit vector orthogonal to A

$2 \cdot 2 + 1 \cdot 2 - 2 \cdot 1 = 4 - 2 - 2 = 0$

$(2, -2, -2)$ $\left(\frac{2}{3}, \frac{2}{3}, -\frac{1}{3}\right)$ or $\left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)$

10 2. In the following diagram carefully draw:

2A A+B A−B 2A+B B−2A projection of A on B



15 3. If $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$ $D = \begin{bmatrix} -1 & 2 \end{bmatrix}$, compute (if possible):

a. $A + B = \begin{bmatrix} 0 & 2 & 3 \\ 3 & 2 & 4 \end{bmatrix}$

b. $A + C = \times$

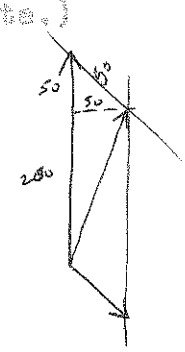
c. $A^t = \begin{bmatrix} 1 & 3 \\ 2 & 1 \\ 1 & 2 \end{bmatrix}$

d. $DA = \underset{2\times 2}{[-1,-2]} \underset{2\times 3}{\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}} = [-7, -4, -5]$

e. $3B = \begin{bmatrix} -3 & 0 & 6 \\ 0 & 3 & 6 \end{bmatrix}$

f. $AC = [\times]$

10 4. If a plane is headed due north at 300 mph and there is a 50 mph wind out of the northwest, what is the ground velocity? (Draw a diagram and estimate.)



$\cancel{50\sqrt{1+4^2} = 50\sqrt{17}}$ $\approx 200+$

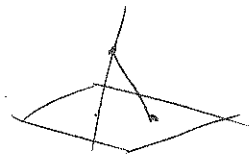
$\tan = \frac{1}{4}$

55

15 5. Find the vector equation for the straight line through (1,2,3) which is perpendicular to the plane $3x-y+z=3$. What is the equation of the set of points on this line which are between the point and the plane?

6

$N = (3,-1,1)$



$X = (1,2,3) + t(3,-1,1)$

$x = 3t+1$
$y = -t+2$
$z = t+3$

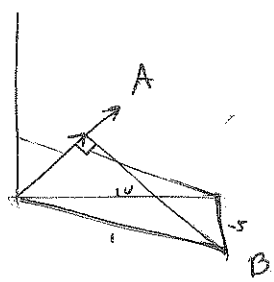
$x =$ same $\quad \cancel{0 \le t \le -\frac{1}{11}}$
$-\frac{1}{11} \le t \le 0$

$3(3t+1) - (-t+2) + t+3 = 3$

$9t+3 + t-2 + t+3 = 3$

$11t = -1 \qquad x = -\frac{3}{11}+1 = \frac{8}{11}$
$t = -\frac{1}{11} \qquad y = 2\frac{1}{11} = \frac{23}{11}$
$z = 3 - \frac{1}{11} = \frac{32}{11}$

10 6. Suppose a sailboater wants to sail northeast (north = (0,1), east = (1,0)), but he would have to sail directly into the wind (impossible). Suppose he sails with velocity vector (10,-5). What is his effective velocity in the direction he wants to go?



$A = (1,1)$

$B = (10,-5)$

$P_{B\,on\,A} = \frac{10-5}{125}(10,-5)$

$= \frac{5}{25}(2,-1)$

$= \frac{1}{5}(2,-1)$

$= \left(\frac{2}{5}, -\frac{1}{5}\right)$

$P_{B\to A} = \frac{5}{2}(1,1)$

$= \left(\frac{5}{2}, \frac{5}{2}\right)$

100

25

10 7. The coefficient matrix for the system of linear equations

$$\begin{aligned} x - 2y + z &= 3 \\ 2x + 2z &= 4 \end{aligned} \quad \text{is} \quad \begin{bmatrix} 1 & -2 & 1 \\ 2 & 0 & 2 \end{bmatrix}$$

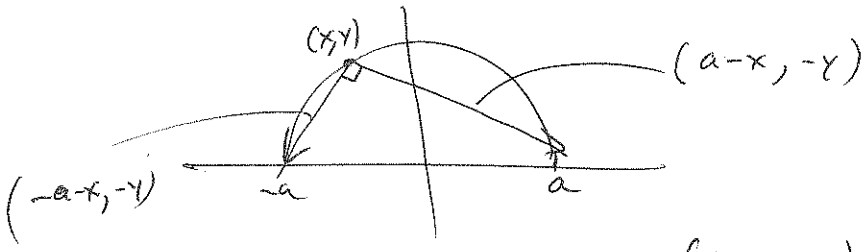
The augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 3 \\ 2 & 0 & 2 & 4 \end{array}\right]$$

Write this system as a matrix equation.

$$\begin{bmatrix} 1 & -2 & 1 \\ 2 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

10 8. Prove: A triangle which is inscribed in a semi-circle (with hypotenuse on the diameter) is a right triangle.



$$(a-x, -y) \cdot (-a-x, -y)$$

$$= -a^2 + x^2 + y^2 = 0$$

$(x,y) = (0,a)$ 5